Multi-domain Boundary Face Method for thermal analysis of gravity dams and its possible implementation by DD

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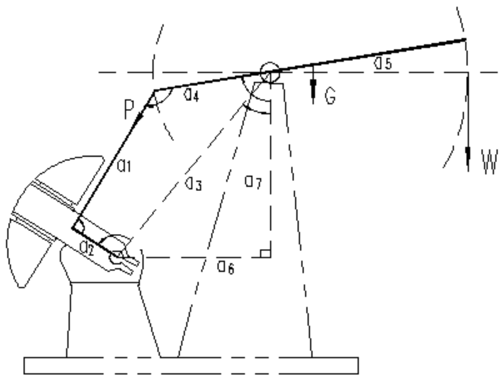
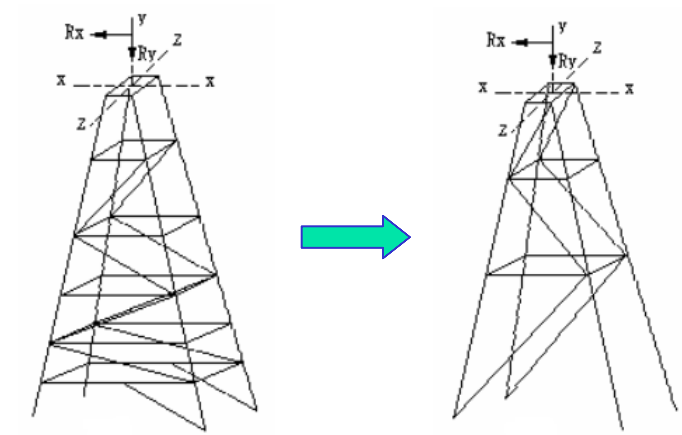
Aug. 3-8 2015

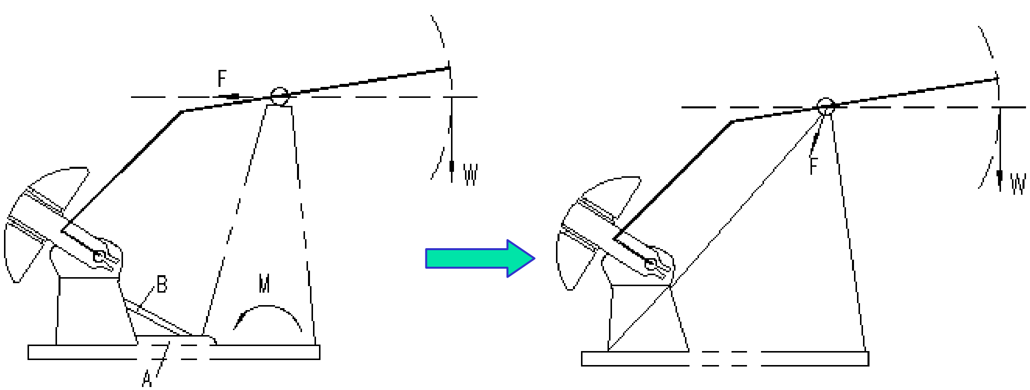
Examples of industrial work

- Sampson post of pumping unit (1991-1992)





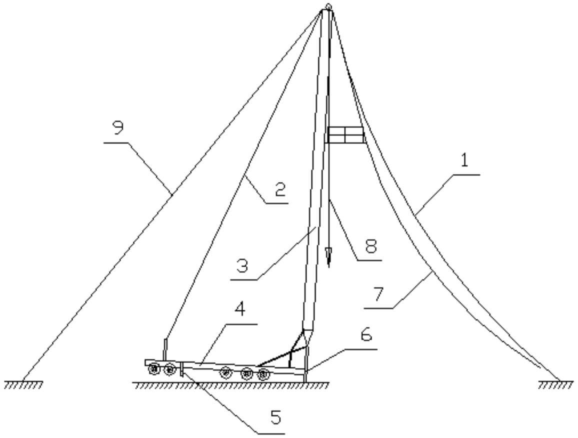
➢ Further improvement

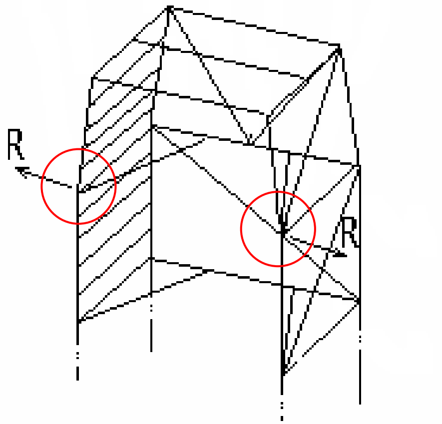


Mass: 2280kg Displacement: 4.4 mm Max. stress: 132 MPa	Mass: 1421kg Displacement: 3.1 mm Max. stress: 58 MPa

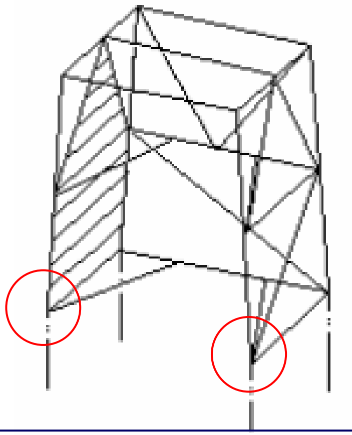
Examples of industrial work (2)

- Workover rig mast (1993-1994)





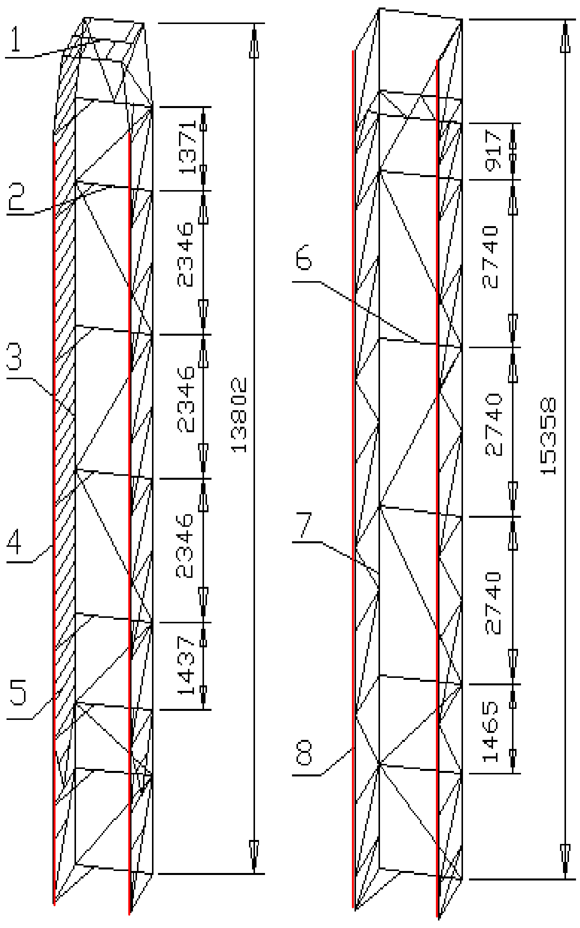
Max. stress: 325 MPa

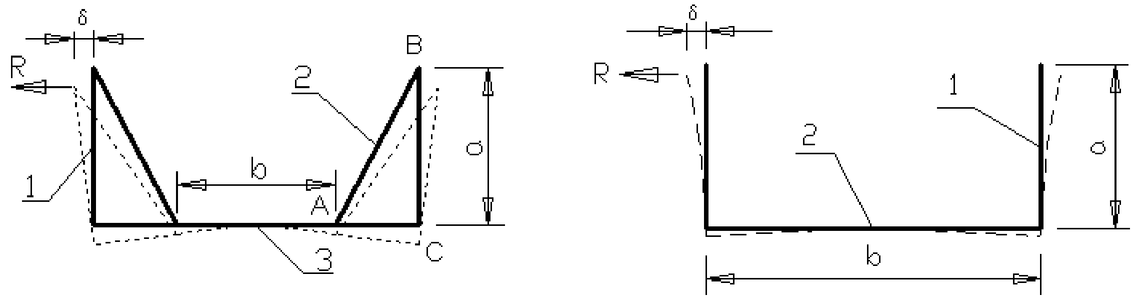


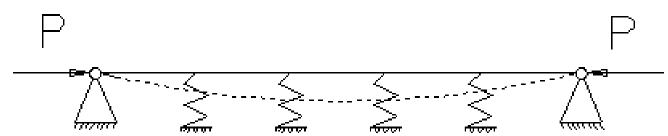
Max. stress: 180 MPa

Examples of industrial work (3)

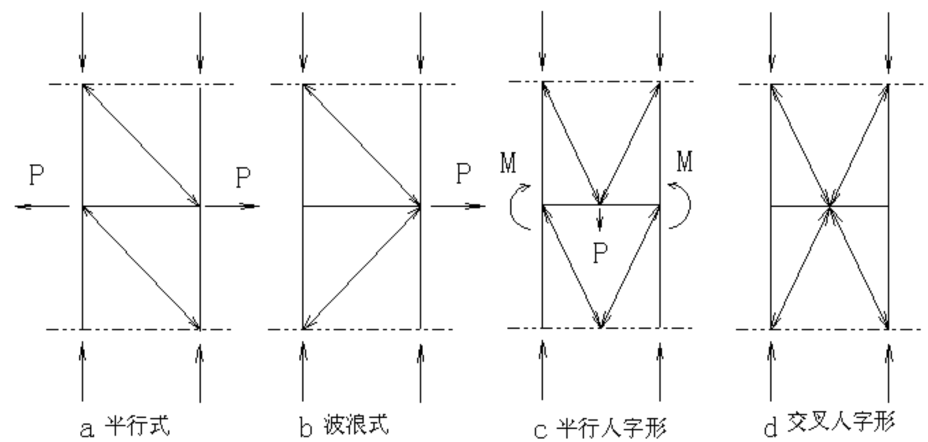
- Further studies





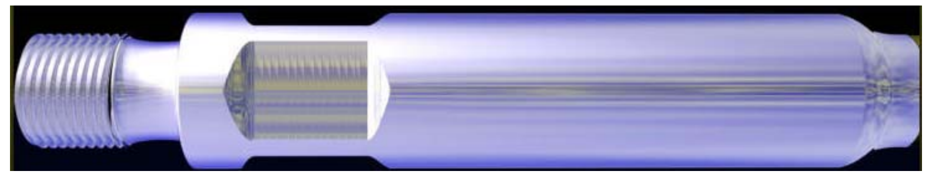


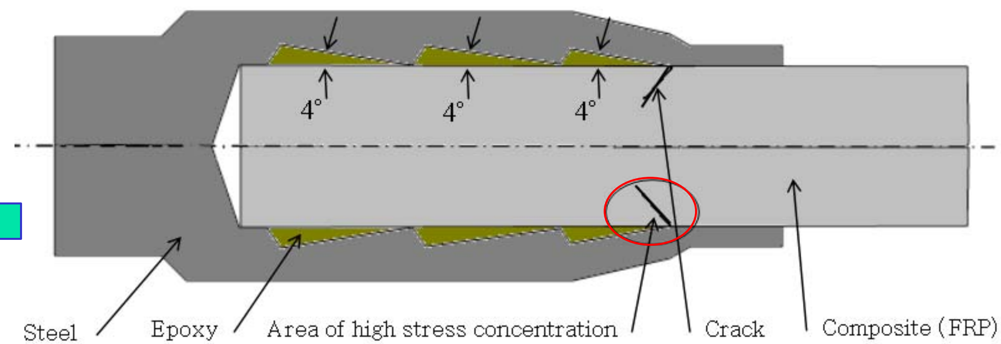
L140X140X12/16Mn → I160×88×6/A3



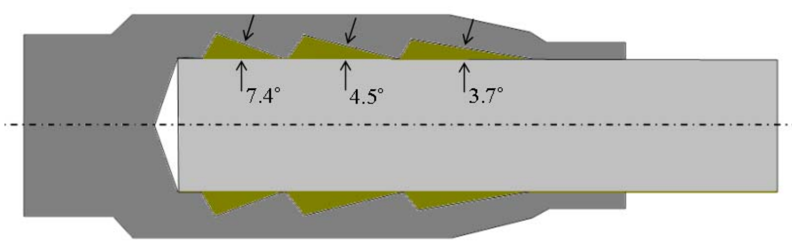
Examples of industrial work (4)

- FRP sucker rod End-fitting (1997-1998)



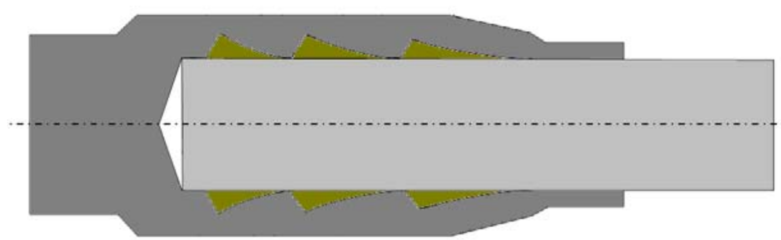


Fatigue cycles: 2.4 million



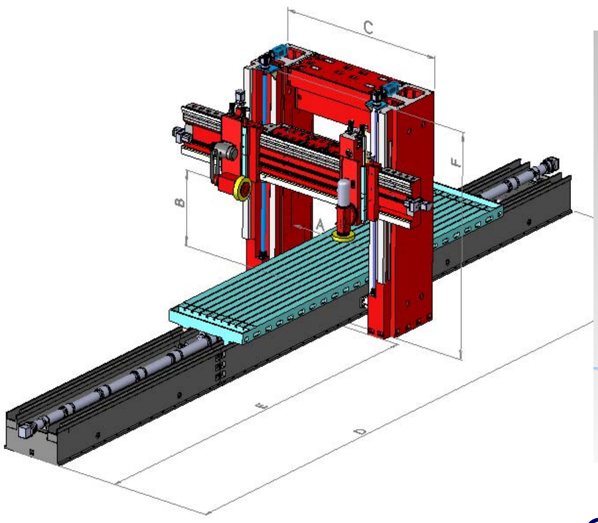
Fatigue cycles: 6.9 million

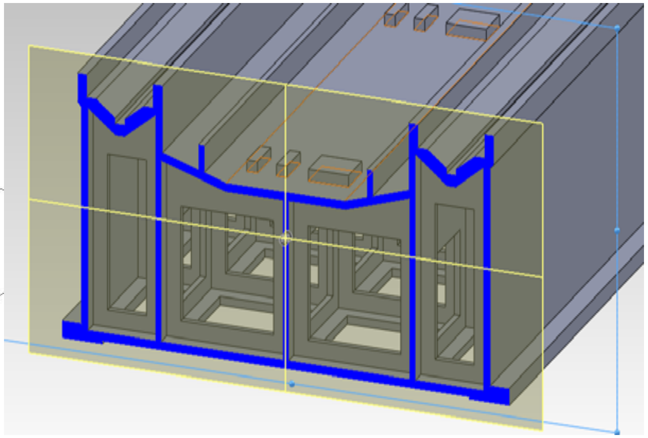
- Idea for further improvement



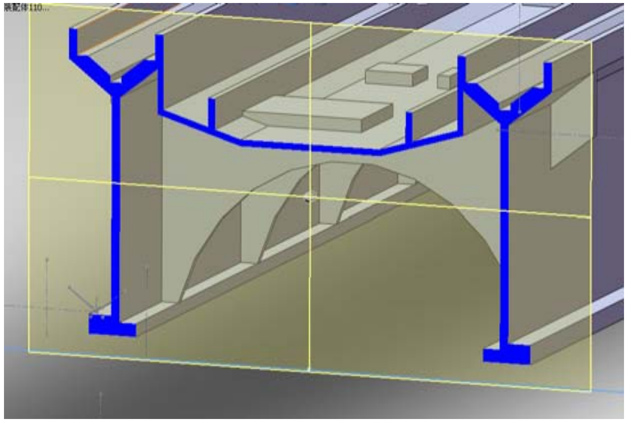
Examples of industrial work (5)

- Gantry Slideway Grinding Machine (2011)





Original Bed Structure

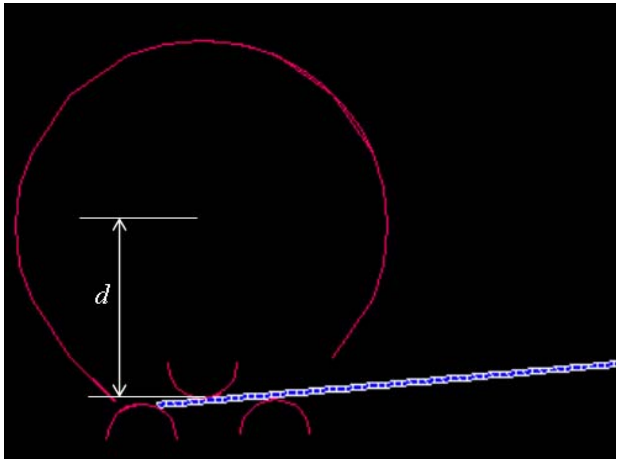


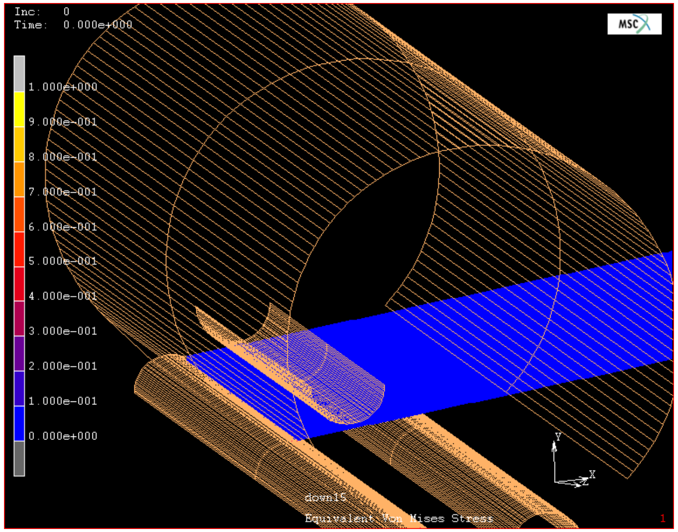
Improved Bed Structure

	Mass (ton)	Mass reduced by	Maximum deflection of Bed	Deflection reduced by	Maximum Deflection of Slideway	Deflection Reduced by
Original	39.598		9.120e-2mm	0	8.1e-2mm	0
Improved	25.274	36%	7.685e-2mm	15.7%	6.5e-2mm	19.7%

Examples of industrial work (6)

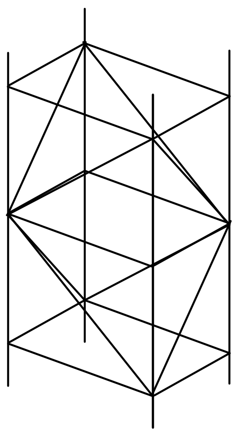
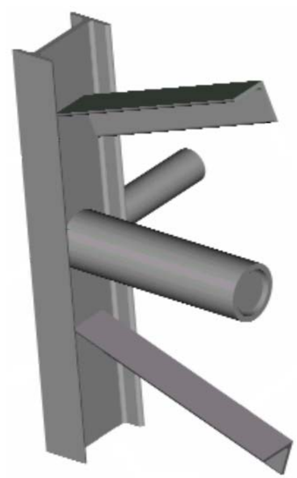
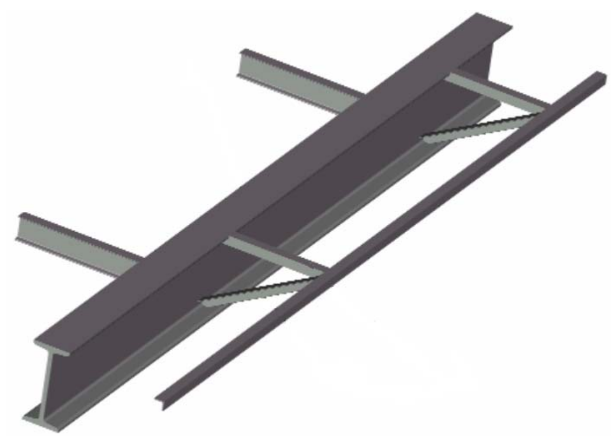
- Forming process of spiral welded pipes (1999)





Difficulties in FEM

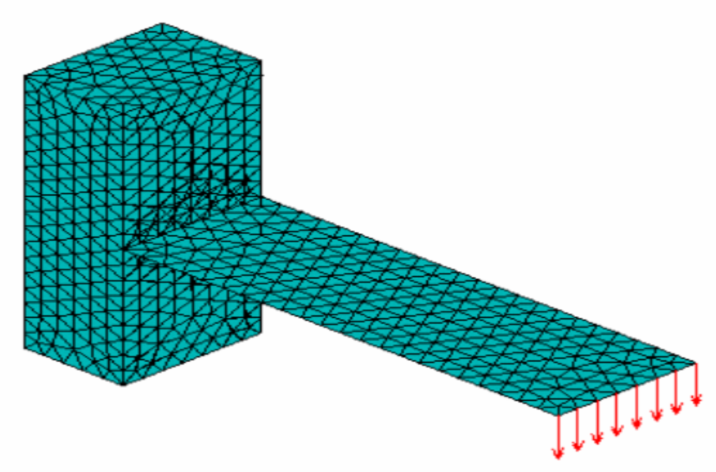
- Continuous parametric model and Discrete model.
  - High quality meshing demanding considerable effort or skill
  - Interaction between CAE and CAD

- Derivable trial function necessary in the weak form
  - Stiffer computational model
  - Contradiction between conforming and nonconforming elements

Difficulties in FEM （2）

- Many kinds of abstract element based on priori assumptions
  - Element performance relies on its shape. Small features are omitted due to connectivity and aspect ratio
  - Accuracy for stresses is of one order lower than displacements
  - New assumptions are required for connecting different kinds of elements, unable to capture local stress
  - Sound and solid training in FEM, rich skills and experiences are a must for a successful user. Analyst and designer are always not the same person



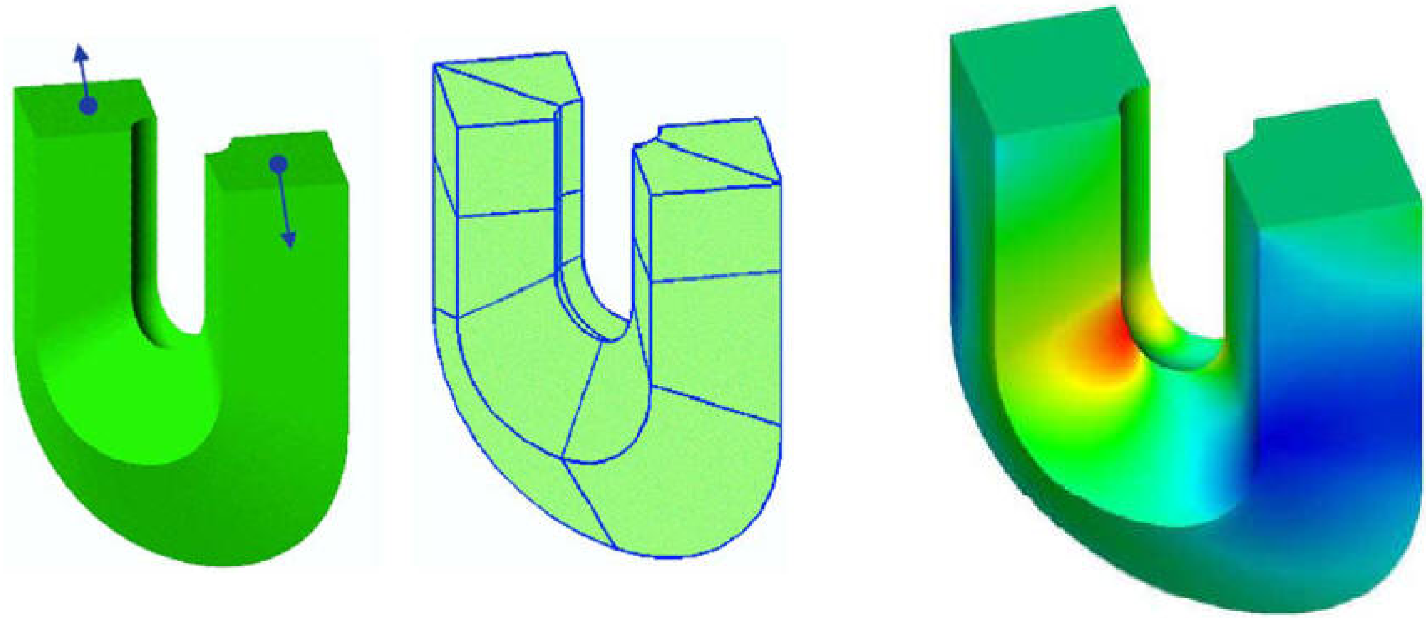
Motivation (5aCAE)

An Ideal CAE tool should have the following Characteristics:

- Automatic meshing and analysis for complicated structures with complex geometries
- Accurately capture local stress concentration at any small sized features of a structure
- Arbitrary geometries and material compositions of structures can be easily handled by seamless interaction with CAD packages
- Accelerated by the fast methods, thus able to perform large-scale computation within due time
- Adaptive solution procedures to guarantee the reliability of the computational results

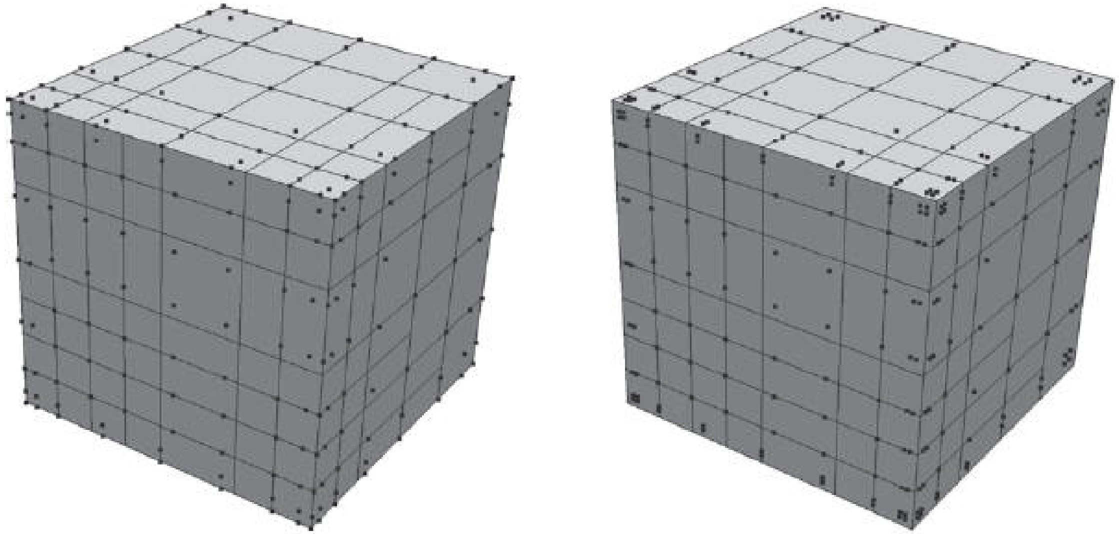
Obstacles in Isogeometric Analysis

- The main idea of the Isogeometric Analysis is to use the same function to approximate the geometry and the physical variables, therefore is can be considered as a "huge" isoparametric element, with the shape function taken from some kinds of splines.



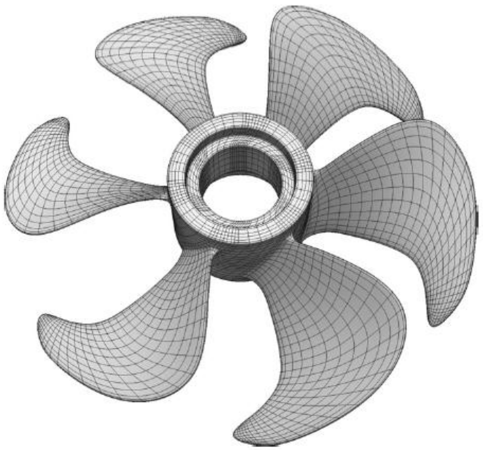
Obstacles in Isogeometric Analysis

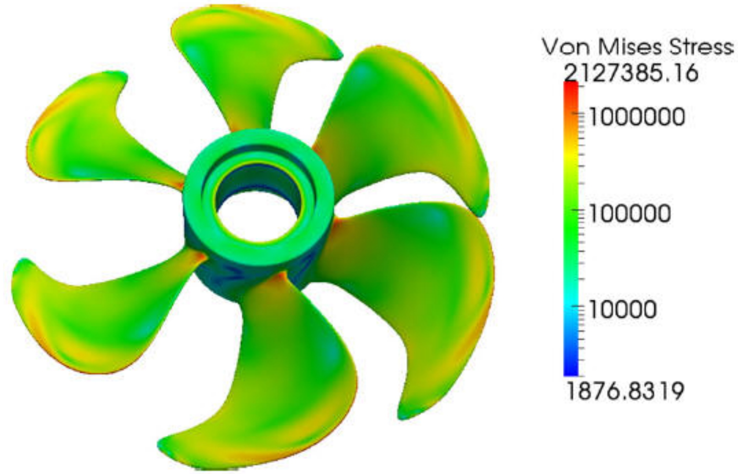
- The geometry of most industrial products consists of bodies with simple surfaces that can be expressed in closed form functions, which are already available in all CAD packages, such as planar, spherical, conic and cylindrical surfaces, etc. Compulsively converting all these surfaces into spline ones is inconvenient, inefficient and also increases the data size drastically.

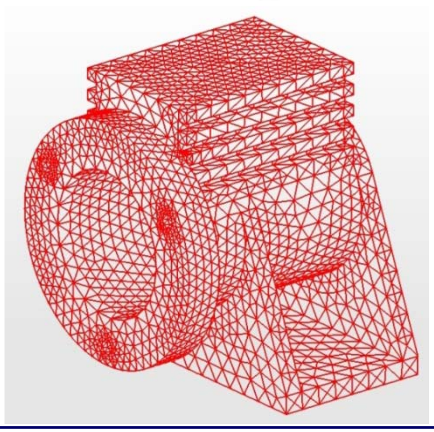


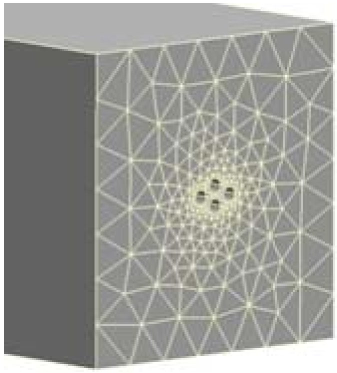
Obstacles in Isogeometric Analysis

- Tackling the gaps between mutually trimmed spline surfaces still remains an obstinate challenge for the Isogeometric Analysis.



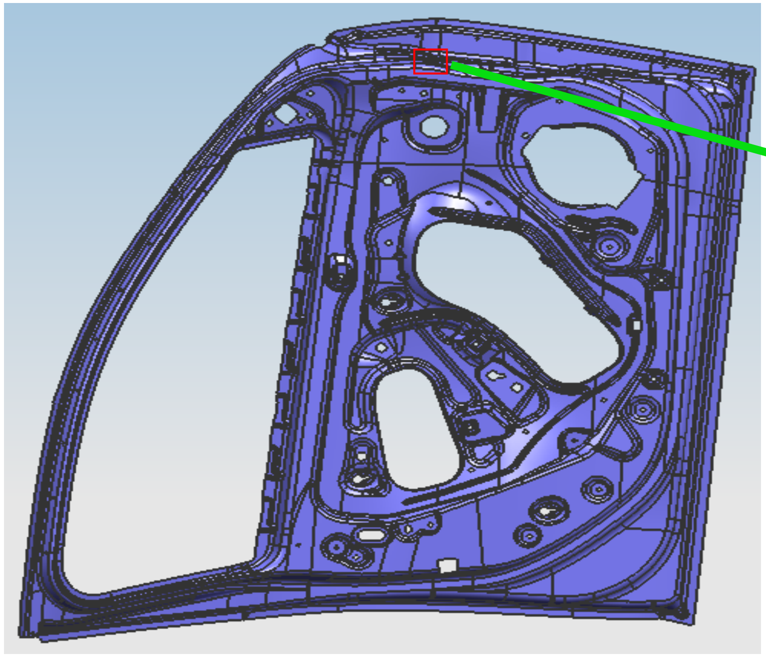


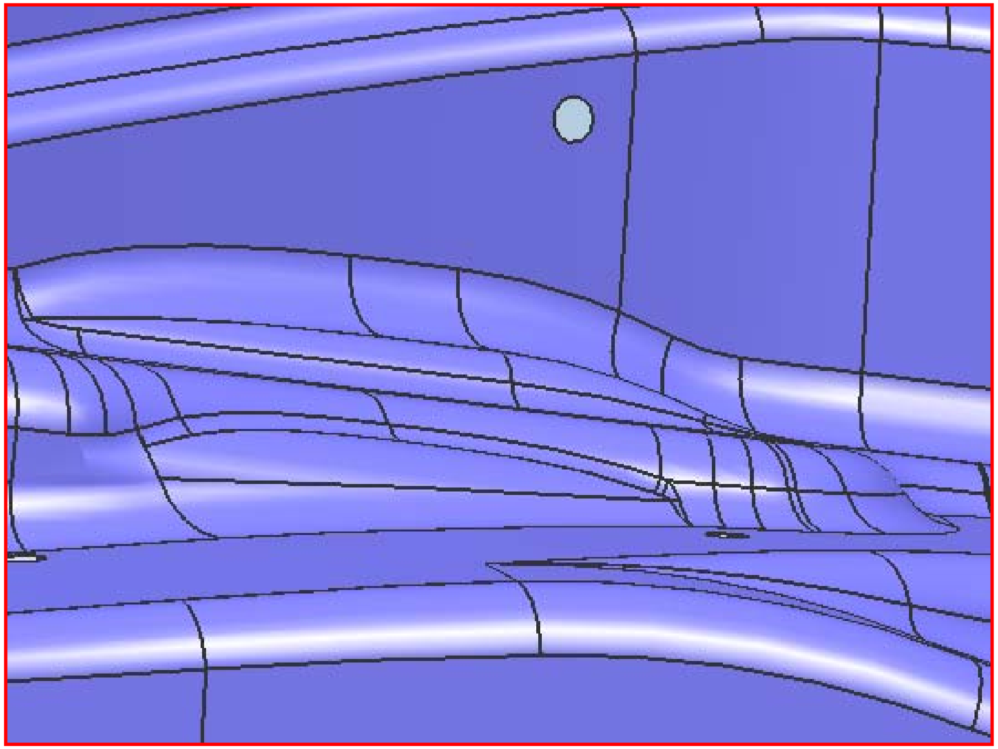




Obstacles in Isogeometric Analysis

- To construct a CAD model using splines that is suitable for Isogeometric Analysis needs the design engineer to be aslo an expert on Computer Graphics.





Review of BIE

- 2D potential problem

$$\nabla^2 u = 0, \quad \forall x \in \Omega$$
$$u = \bar{u}, \quad \forall x \in \Gamma_u$$
$$\frac{\partial u}{\partial n} \equiv q = \bar{q}, \quad \forall x \in \Gamma_q$$

- The equivalent weak form

$$\int_\Omega v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) dxdy + \int_{\Gamma_q} \bar{v}\left(k\frac{\partial u}{\partial n} - \bar{q}\right) d\Gamma = 0$$

- Once integration by part, FEM formulation

$$\int_\Omega \left(\frac{\partial v}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial u}{\partial y}\right) dxdy - \int_{\Gamma_q} v\bar{q}\, d\Gamma = 0$$

- Twice integration by part, BIE formulation

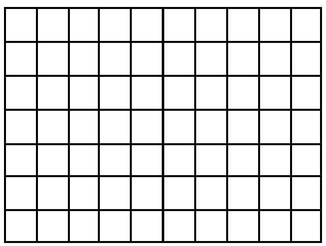
$$\int_\Omega u\nabla^2 v\, d\Omega - \int_\Gamma \left(\frac{\partial v}{\partial n}u - v\frac{\partial u}{\partial n}\right) d\Gamma = 0$$

- Contradiction between conforming and nonconforming elements
- Locking problems: membrane locking, volumetric locking, shear locking etc.
- Reduced integration and hourglass modes
- Accuracy of fluxes is one order lower than that of potential

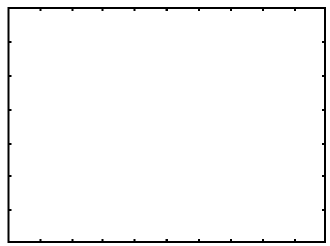
Advantages of BIEM

- Easy mesh generation and modification

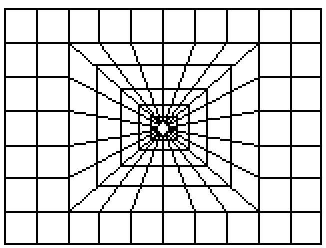
Adding a hole



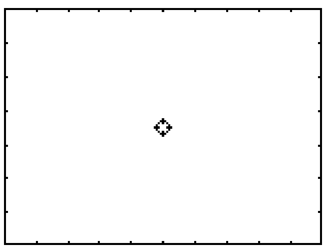
Domain type



Boundary type



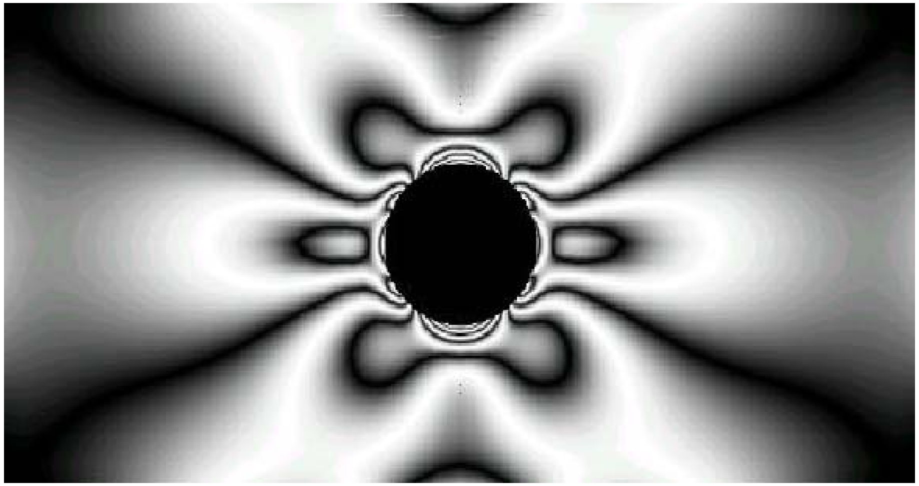
Domain type



Boundary type

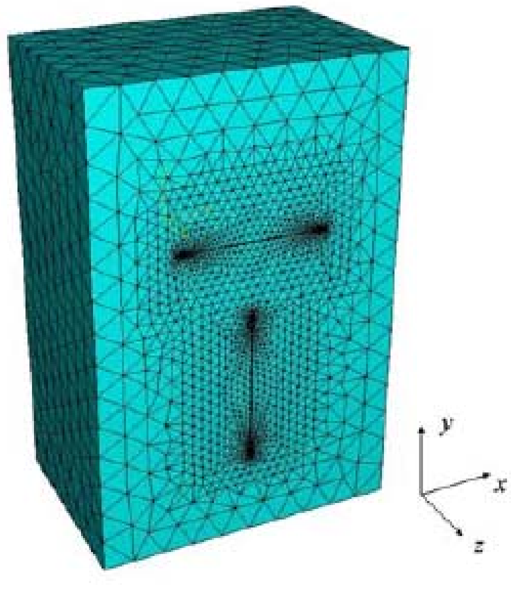
Potential to make direct use of a body's parametric representation through Brep data of CAD packages

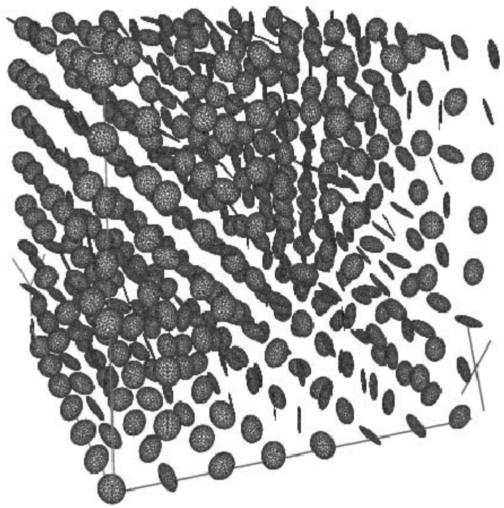
- High accuracy for local stress concentration



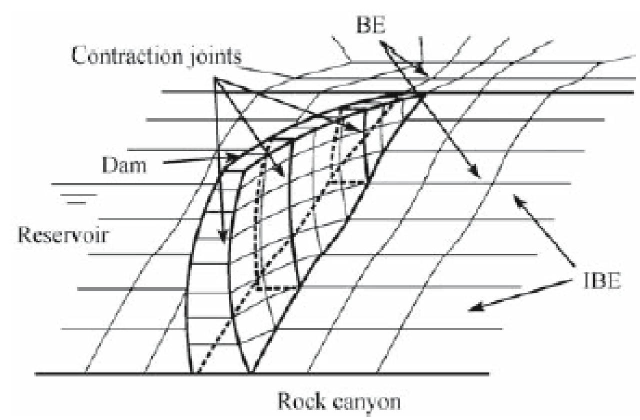
Advantages of BIEM

- Suitable for solving singular problems





- Suitable for solving problems involving infinite domains



Complete coupled system of dam-canyon-reservoir

Disadvantages of Traditional BEM

- Implementation does not take full advantages of the BIE that the BIE has real potential make direct use of the Brep data of a solid, and hence unify CAE and CAD in a unique framework.

- Dense and unsymmetrical coefficient matrices

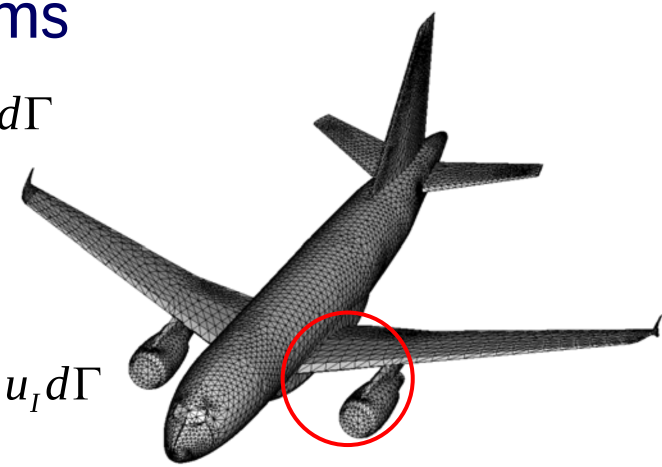
  - Memory complexity $O(N^2)$
  - CPU complexity Direct solver： $O(N^3)$
    Iterative solver： $O(N^2)$

- Singular and nearly singular integration involves complex mathematical operations

Boundary Face Method

- The self-regular BIE for potential problems

$$0=\int_\Gamma (u(\mathbf{s})-u(\mathbf{y}))q^s(\mathbf{s},\mathbf{y})\,d\Gamma-\int_\Gamma q(\mathbf{s})u^s(\mathbf{s},\mathbf{y})\,d\Gamma$$



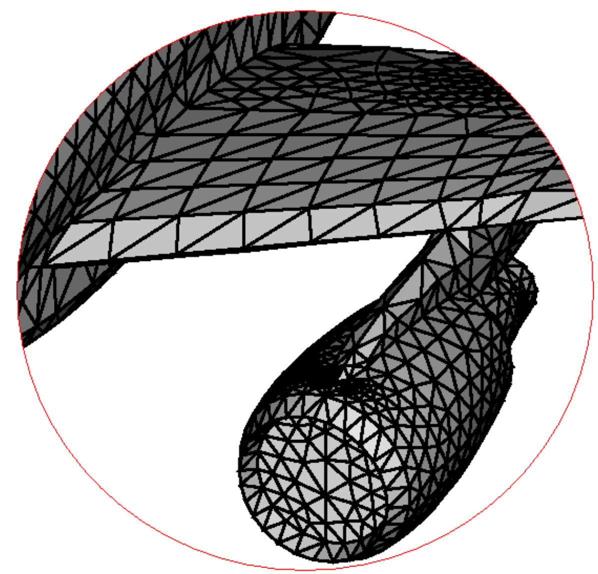
- The discretized form by elements

$$0=\sum_{j=1}^{N_e}\int_{\Gamma_j} u^s(\mathbf{s},\mathbf{y})\sum_{I=1}^{N_p}\Phi_I(\mathbf{s})\,q_I\,d\Gamma-\sum_{j=1}^{N_e}\int_{\Gamma_j} q^s(\mathbf{s},\mathbf{y})\sum_{I=1}^{N_p}(\Phi_I(\mathbf{s})-\Phi_I(\mathbf{y}))\,u_I\,d\Gamma$$

In standard BEM,

elements are used to

- **facilitate boundary integration**
- **interpolate boundary variables**
- **approximate the geometry**



Boundary Face Method (2)

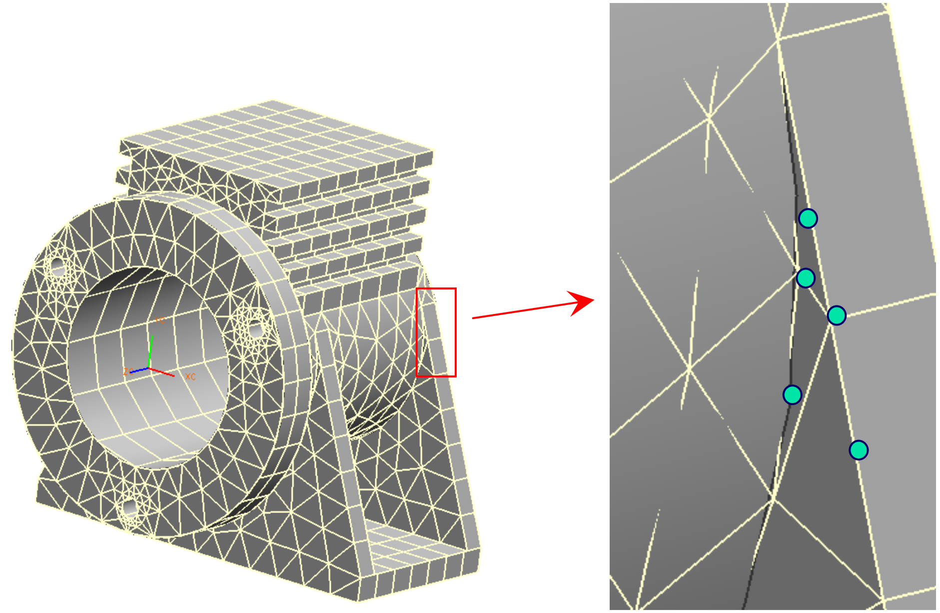
- The discretized form of BIE in BFM

$$0=\sum_{j=1}^{N_c}\int_{\Gamma_j} u^s(\mathbf{s},\mathbf{y})\sum_{I=1}^{N}\Phi_I(\mathbf{s})\,\hat{q}_I\,d\Gamma-\sum_{j=1}^{N_c}\int_{\Gamma_j} q^s(\mathbf{s},\mathbf{y})\sum_{I=1}^{N}(\Phi_I(\mathbf{s})-\Phi_I(\mathbf{y}))\,\hat{u}_I\,d\Gamma$$

In the BFM,

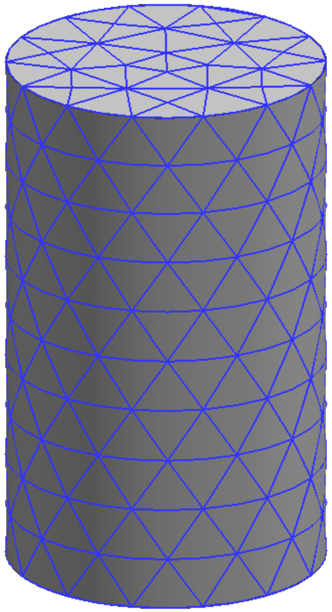
elements are used to

- **facilitate boundary integration, only**
- **Shape functions are separated from the elements**
- **The exact geometry is preserved**

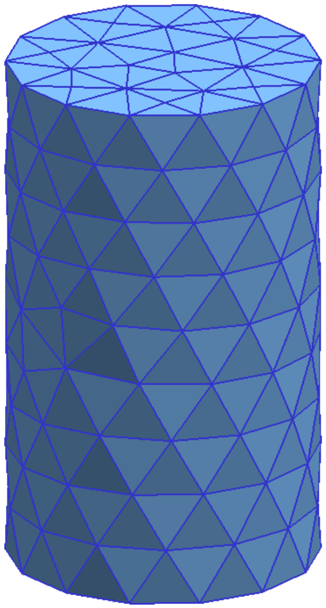


Boundary Face Method (3)

- Discretization of a cylinder with linear triangular elements

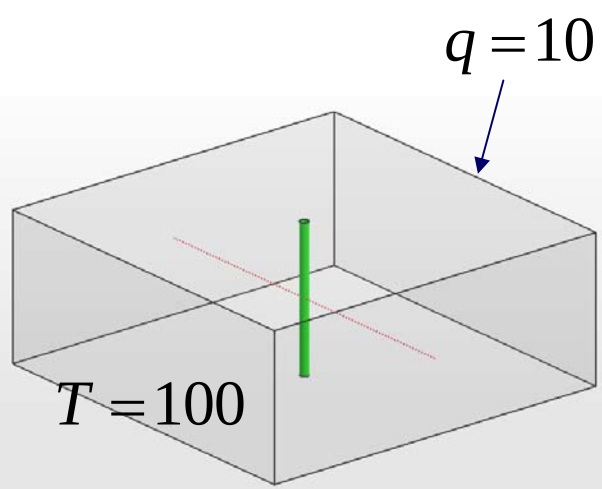


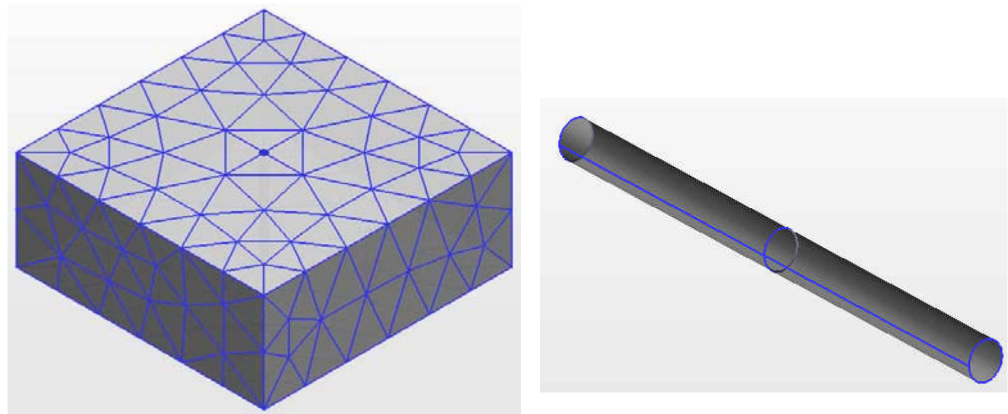
BFM model



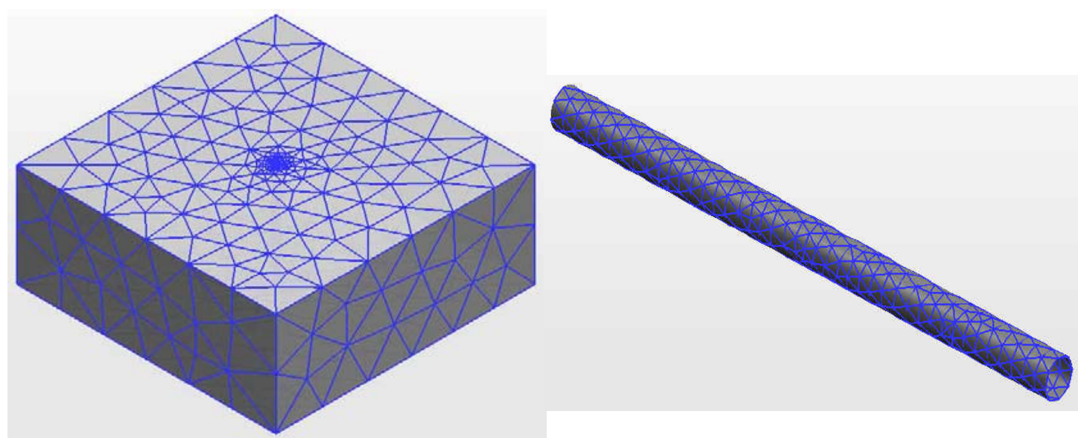
BEM model

Boundary Face Method (5)





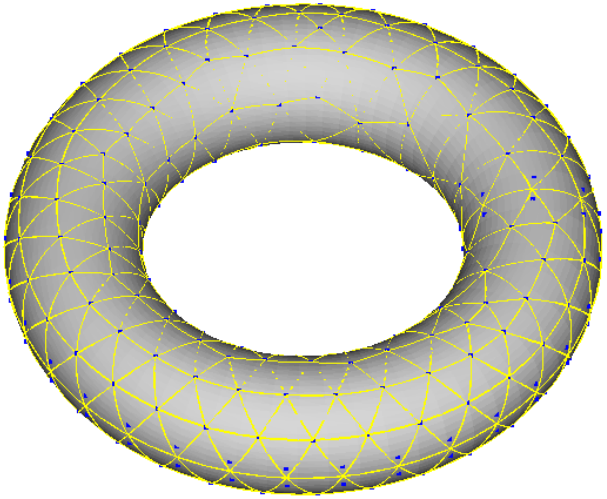
BFM



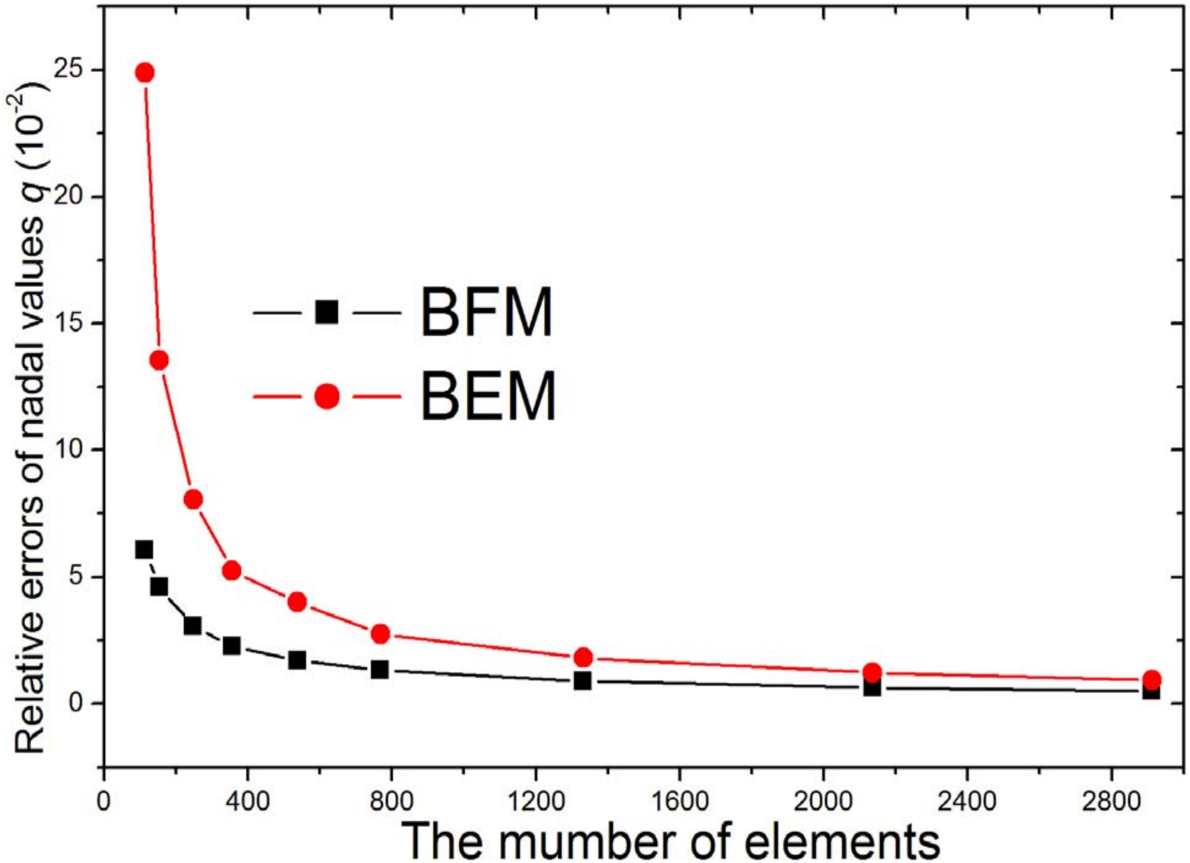
BEM

Boundary Face Method (4)

- Comparison with BEM



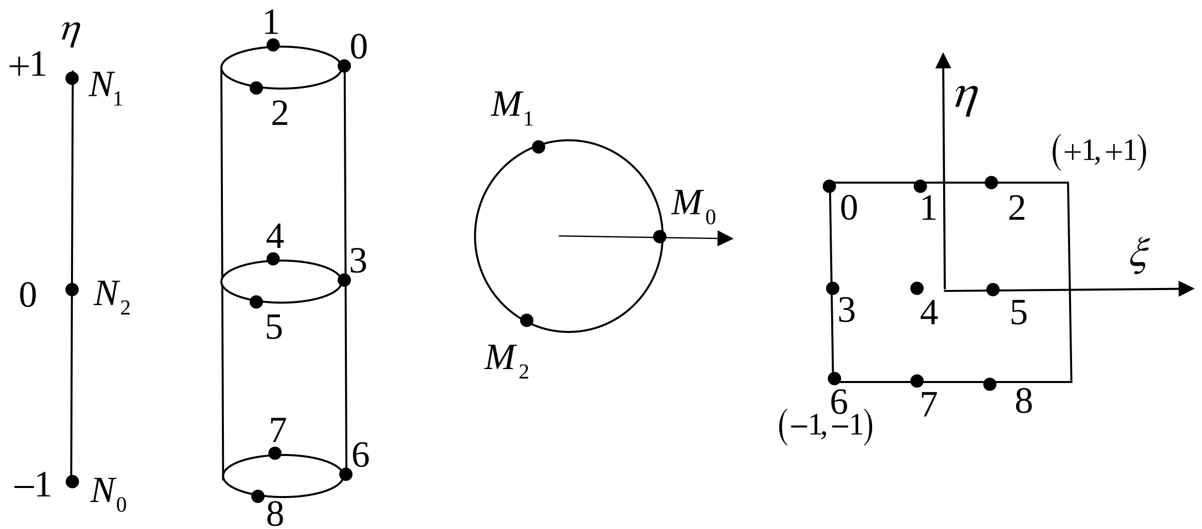
Surface mesh
(538 elements)



Comparison between BFM and BEM

- Cooling water pipes

  - **Cylinder Element**



$$N_0 = -\frac{1}{2}\eta(1-\eta)$$

$$N_1 = \frac{1}{2}\eta(1+\eta)$$

$$N_2 = (1+\eta)(1-\eta)$$

$$M_0(\theta) = \frac{1}{3} + \frac{2}{3}\cos\theta$$

$$M_1(\theta) = \frac{1}{3} + \frac{\sqrt{3}}{3}\sin\theta - \frac{1}{3}\cos\theta$$

$$M_2(\theta) = \frac{1}{3} - \frac{\sqrt{3}}{3}\sin\theta - \frac{1}{3}\cos\theta$$

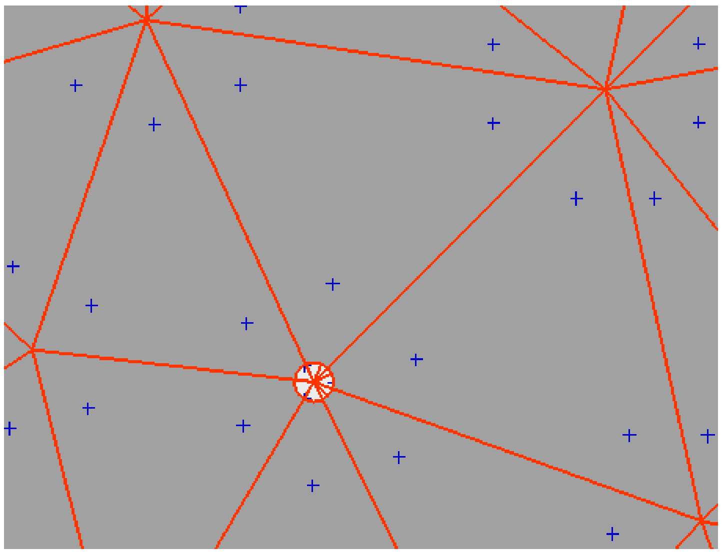
$$\phi_6 = M_0 N_0, \phi_7 = M_1 N_0, \phi_8 = M_2 N_0$$

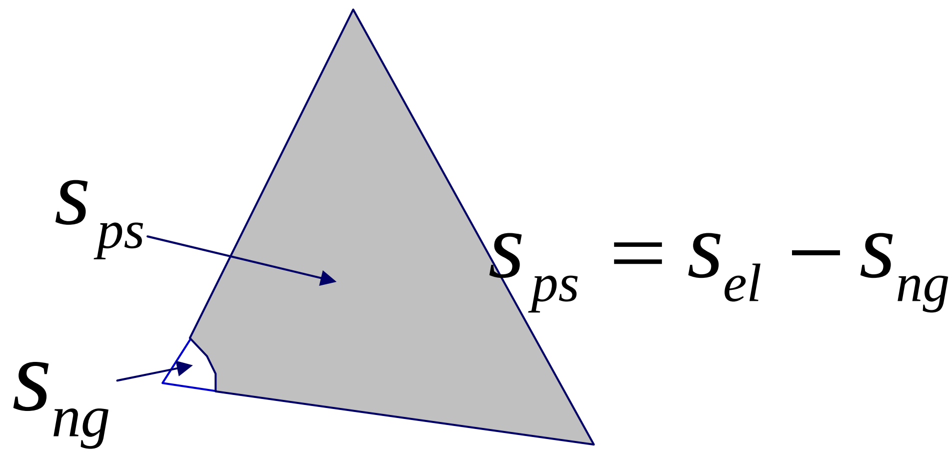
$$\phi_0 = M_0 N_1, \phi_1 = M_1 N_1, \phi_2 = M_2 N_1$$

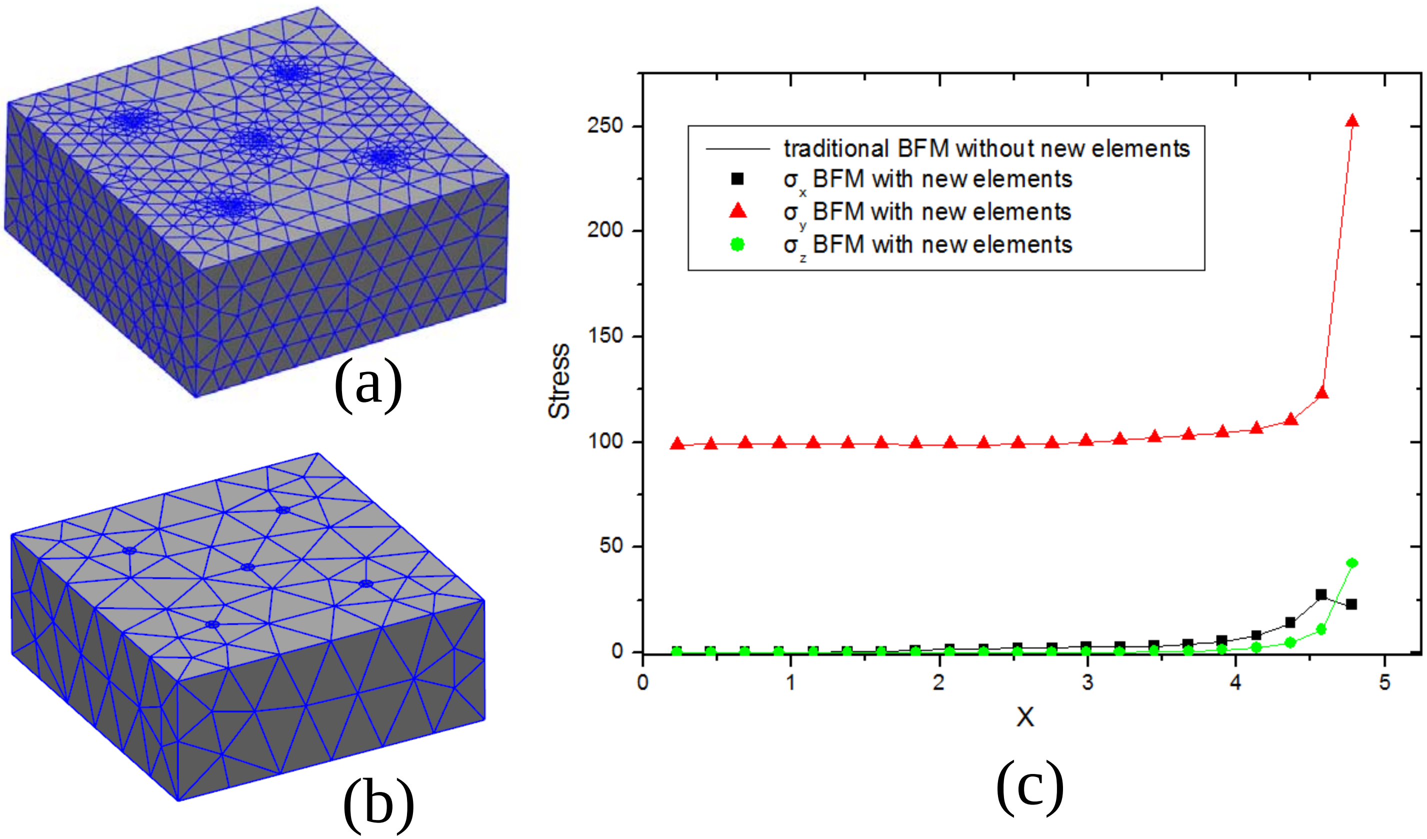
$$\phi_3 = M_0 N_2, \phi_4 = M_1 N_2, \phi_5 = M_2 N_2$$

Boundary Face Method (6)

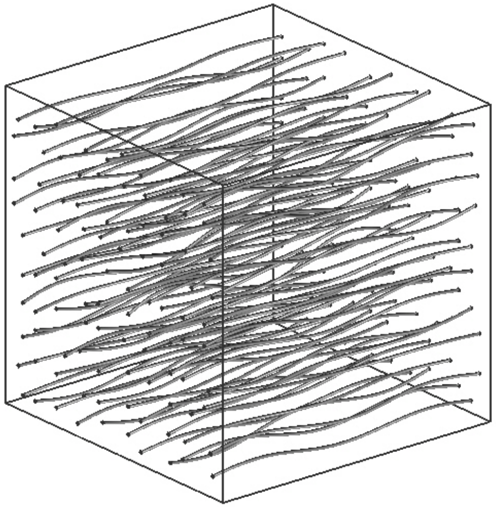
- **Triangle Element with negative parts**

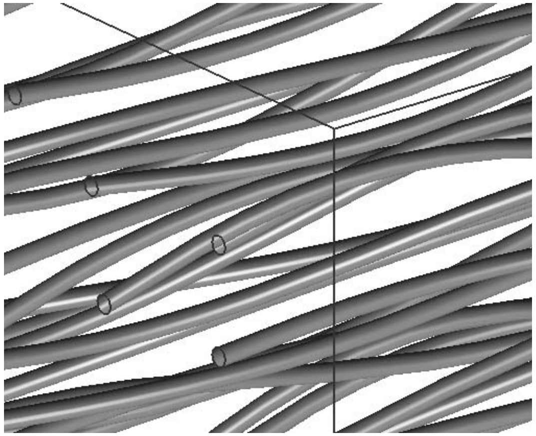




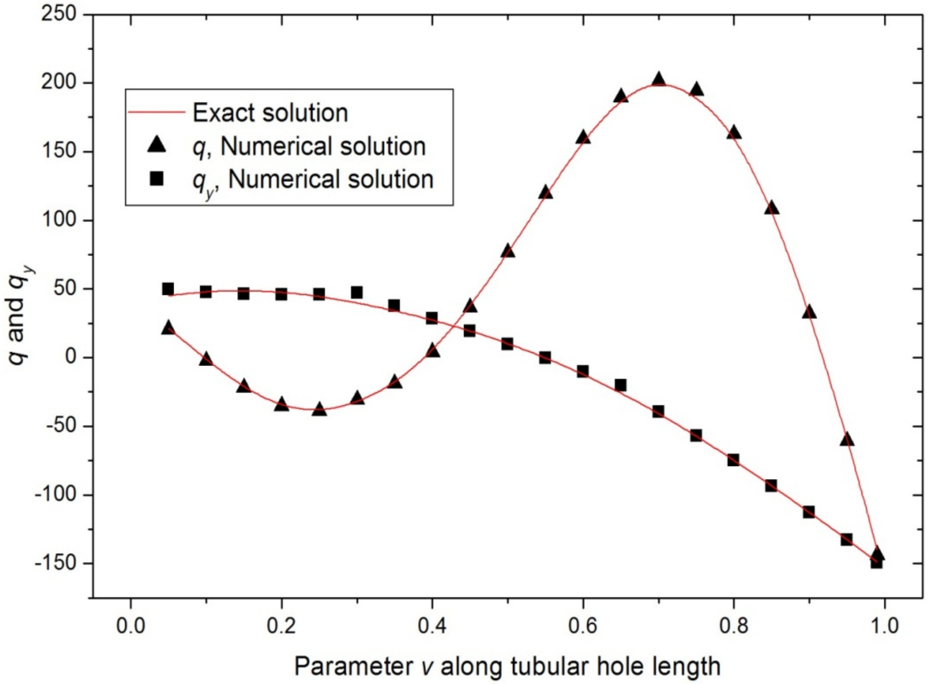

(a)
(b)
(c)
traditional BFM without new elements
σx BFM with new elements
σy BFM with new elements
σz BFM with new elements
Stress
X
0
50
100
150
200
250
0
1
2
3
4
5

- **A Cuboid with many randomly spaced curved pipes**

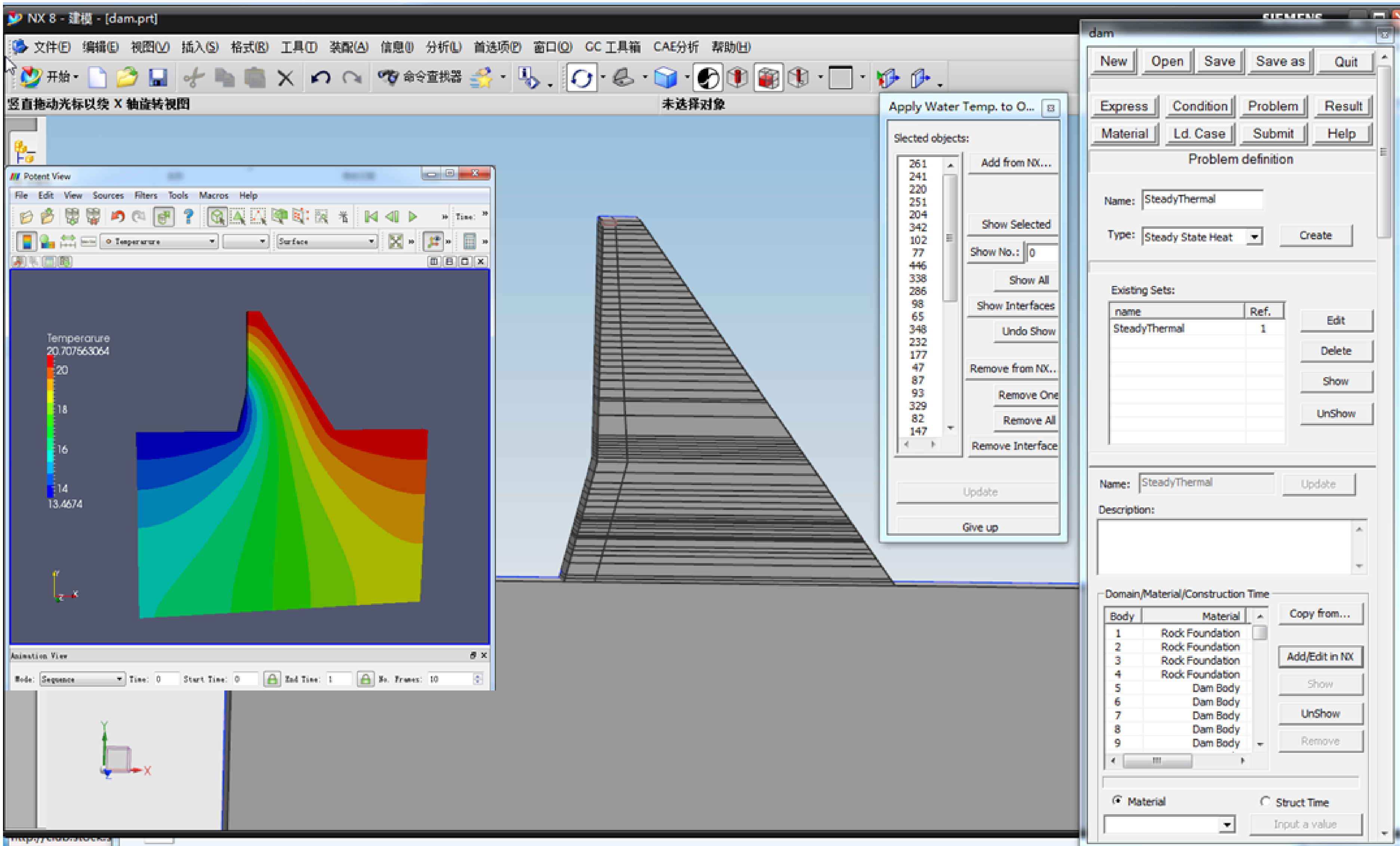




Elements: 1994;   Nodes : 7782

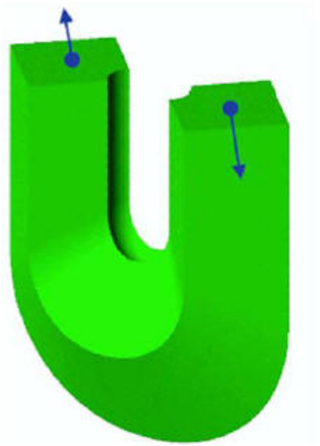


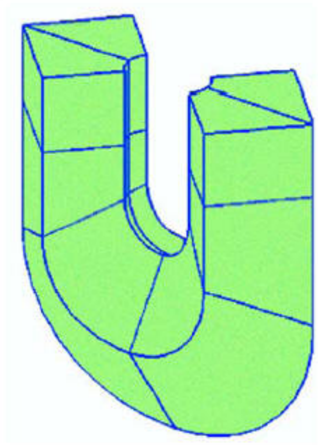
Integrate the BFM into UG-NX

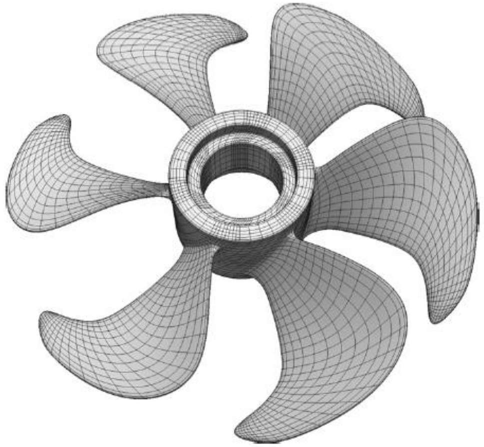


■ Boundary Face Method (9)

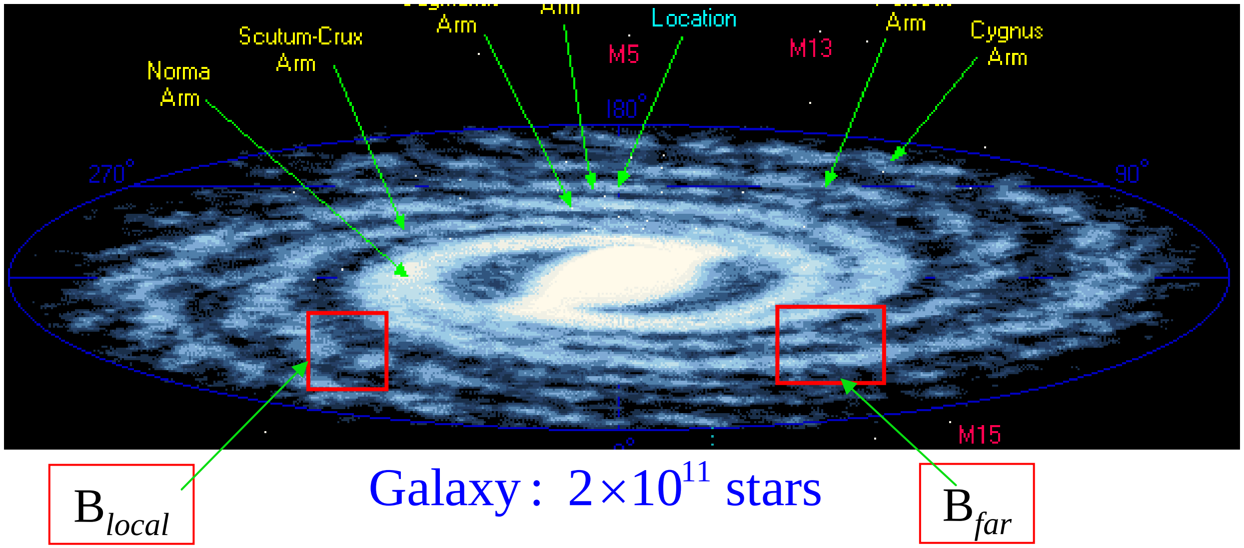
- The BFM combines the BIE and Computer graphics, implementing the BIE analysis directly on a CAD model.
- The BFM is a general framework of BIEM. It is an extension of BEM, and hence includes the BEM, namely any BEM code can be easily merged into the BFM.
- Any Isogeometric Analysis model can be analyzed by the BFM, because it is a kind of CAD model.







Breakthrough in N-Body Problem



Galaxy： 2×10^{11} stars

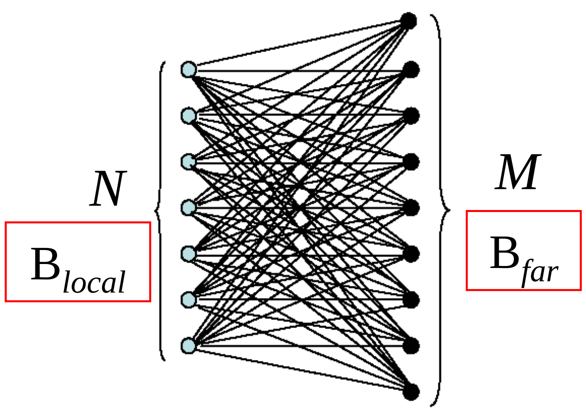
$$F_i = \sum_{j=1}^{N} G m_i m_j / r_{ij}^2$$

$$6\times(2\times10^{11})^2 \approx 2.4\times10^{23}$$

$$t = 7.6\times10^6 \text{ year} \ (10^8 \, Flops)$$

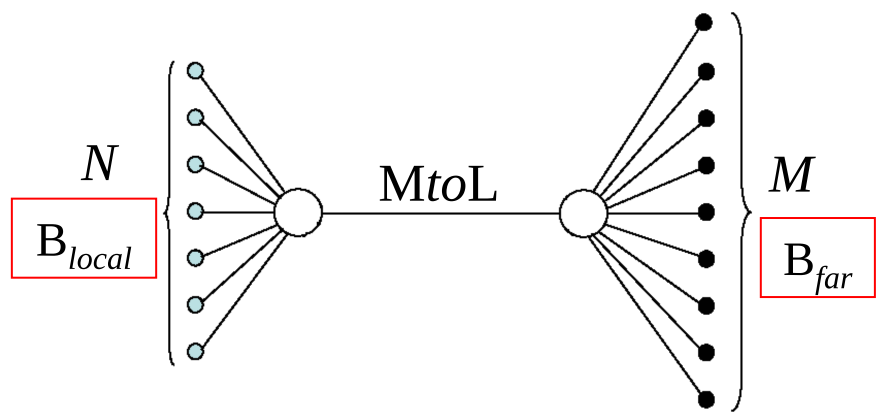
FMM：

$$\text{t} = 3.3 \text{ Hours} \ (10^8 \, Flops)$$



Straightforward

Total number of operations O(NM)

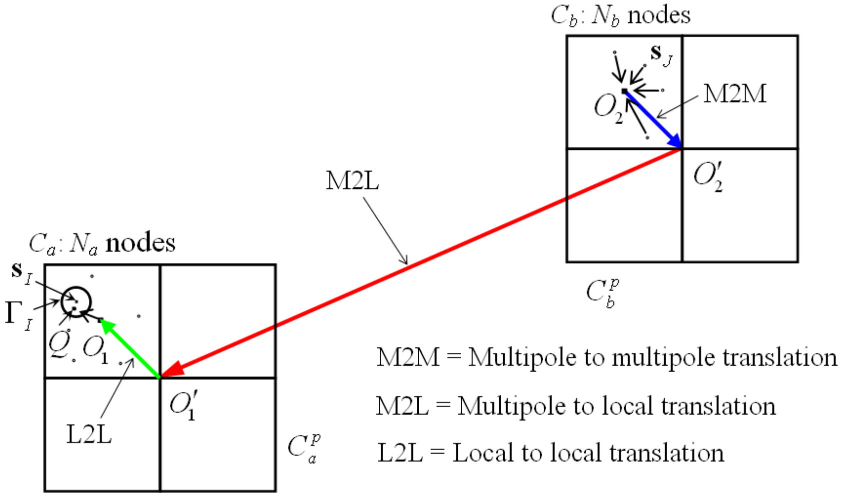


Multipole expansion

Total number of operations O(N+M)

Breakthrough in N-Body Problem

- **Fast Multipole Method**

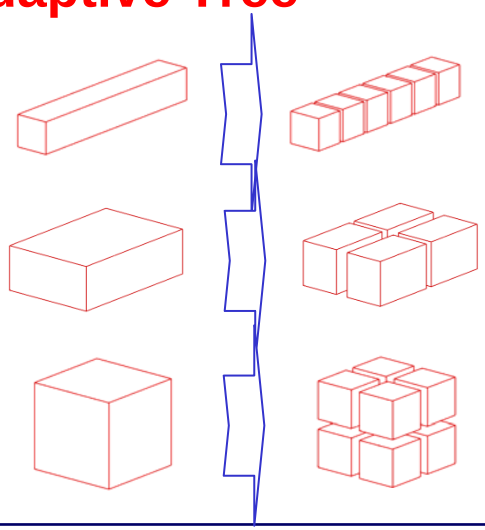


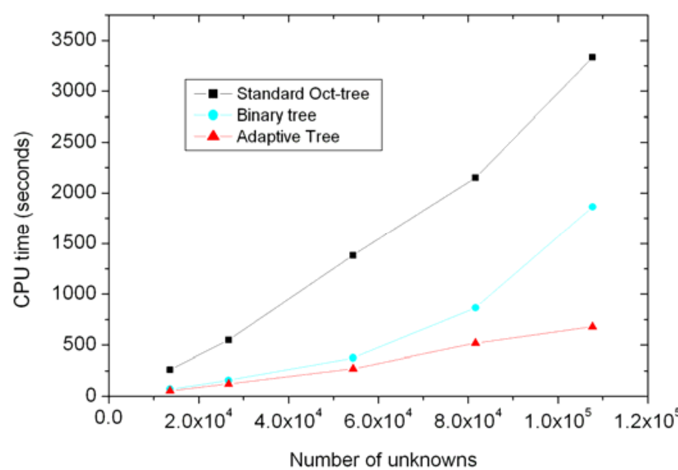
$$M_{n'}^{m'}(Q_2') = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} R_n^m(\overrightarrow{O_2'O_2})\, M_{n-n'}^{m-m'}(Q_2)$$

$$L_n^m(O_1') = \sum_{n'=0}^{\infty}\sum_{m'=-n'}^{n'} (-1)^n\, \overline{S_{n+n'}^{m+m'}}(\overrightarrow{O_1'O_2'})\, M_{n'}^{m'}(Q_2')$$

$$L_{n'}^{m'}(O_1) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} R_{n-n'}^{m-m'}(\overrightarrow{O_1'O_1})\, L_n^m(Q_1')$$

- **Adaptive Tree**





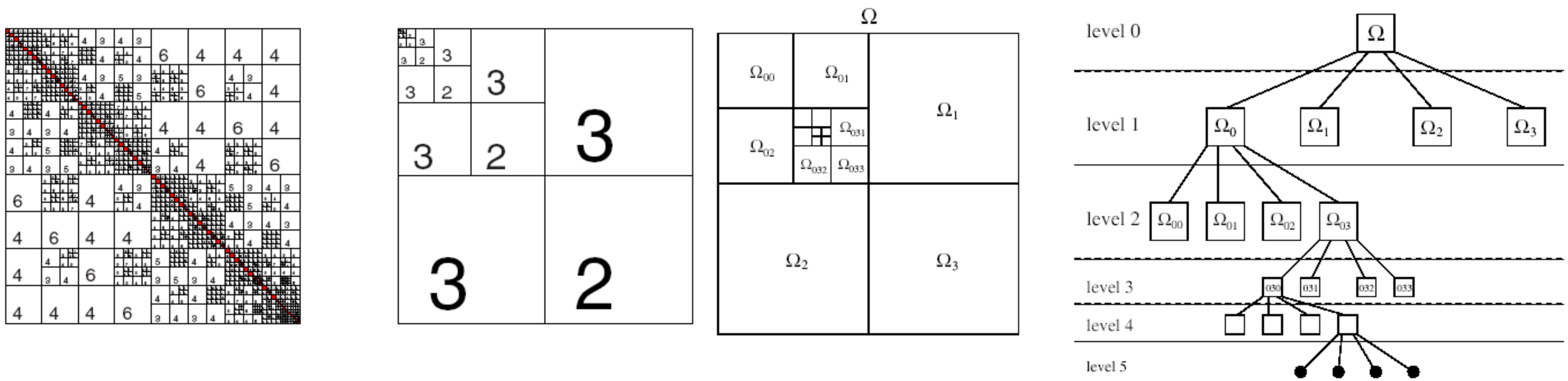
By JM. Zhang

J. Comput. Phys.

Vol. 226 (2007), pp. 17-28

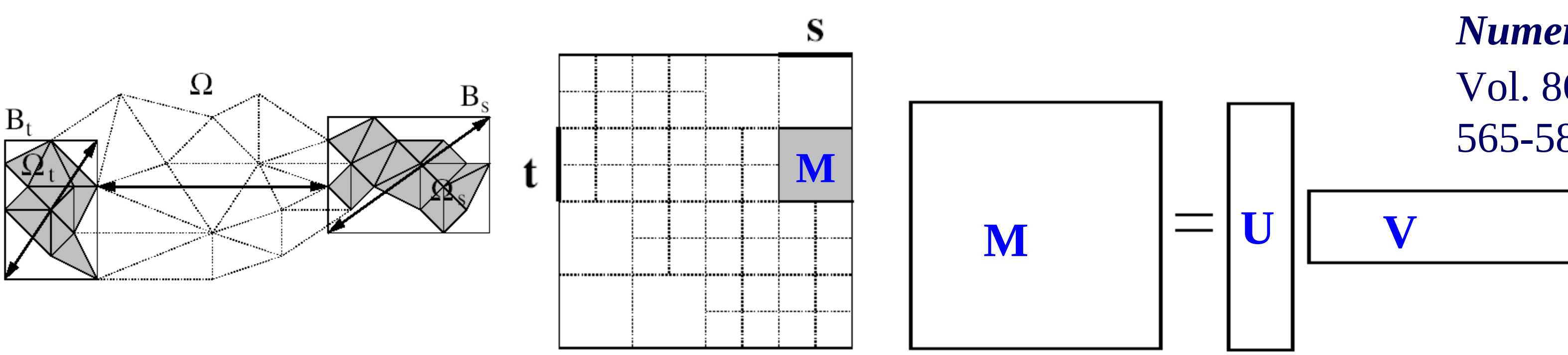
Breakthrough in N-Body Problem

- **Hierarchical matrix (H -matrix)**



- **Adaptive Cross Approximation(ACA)**

By M. Bebendorf
Numer. Math.
Vol. 86 (2000), pp. 565-589



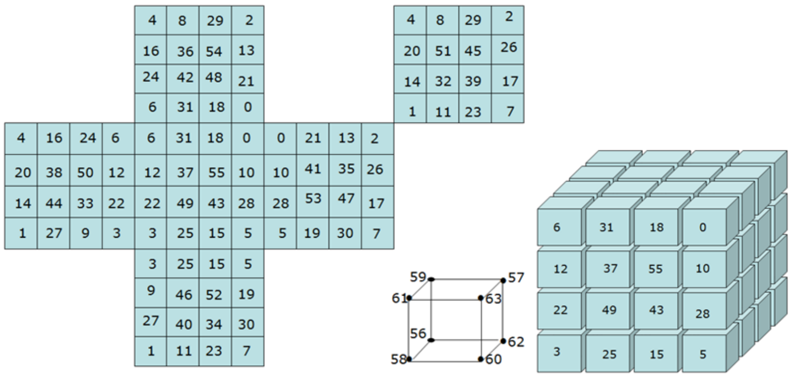
Geometric Cross Approximation(GCA)

By JM. Zhang

EABE

Vol. 37 (2013), pp. 1668-1673

$$\kappa(P,Q) \approx \{\kappa(P,Q_1) \quad \ldots \quad \kappa(P,Q_k)\} \begin{bmatrix} \kappa(P_1,Q_1) & \cdots & \kappa(P_1,Q_k) \\ \vdots & & \vdots \\ \kappa(P_k,Q_1) & \cdots & \kappa(P_k,Q_k) \end{bmatrix}^{-1} \begin{Bmatrix} \kappa(P_1,Q) \\ \vdots \\ \kappa(P_k,Q) \end{Bmatrix}$$



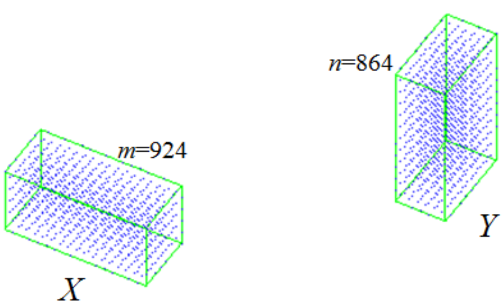
$$\tilde{\mathbf{A}} = CD^{-1}R$$

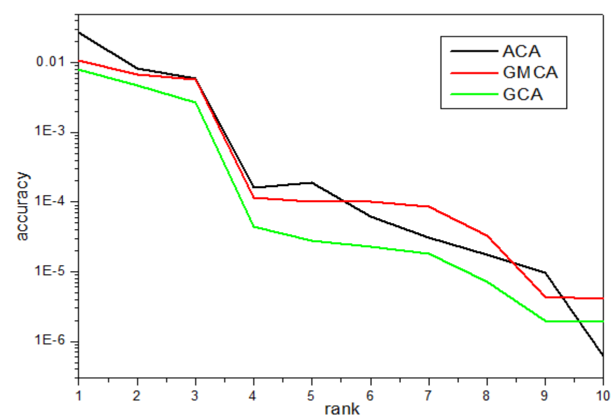
$$C = \mathbf{A}(I,\hat{J}) \qquad R = \mathbf{A}(\hat{I},J) \qquad D = \mathbf{A}(\hat{I},\hat{J})$$

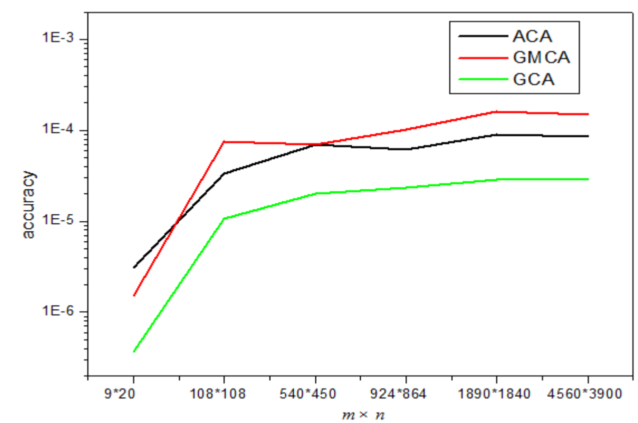
$$I \equiv \{1, \ \ldots, m\} \qquad J \equiv \{1, \ \ldots, n\}$$

$$D = u\Sigma v \qquad D^{-1} = v^T \Sigma^{-1} u^T$$

$$\boldsymbol{U} = Cv^T \qquad \boldsymbol{V}^T = \Sigma^{-1} u^T R$$







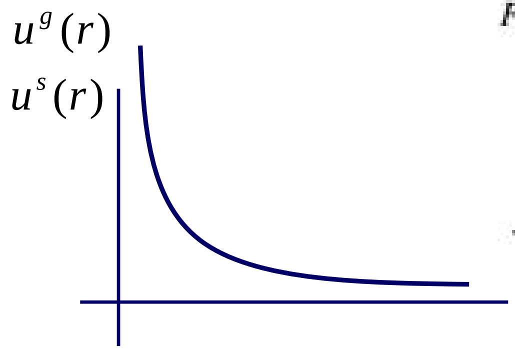
- The a-priori choice of pivots can be advantageous also in H²-matrix

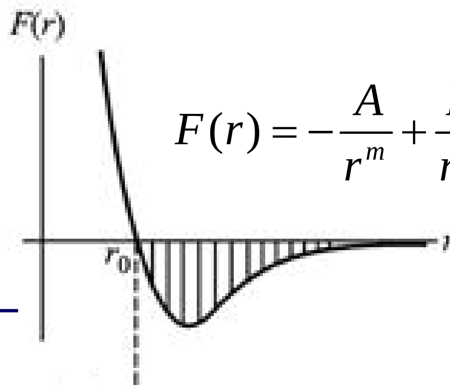
Breakthroughs in BIEM

$$u^{g} = \frac{Gm_1m_2}{r}$$

$$u^{s}(r) = \frac{1}{4\pi}\frac{1}{r(P,Q)}$$

$$u^{s}_{ij}(r) = \frac{1}{16\pi(1-v)Gr}\left[(3-4v)\delta_{ij} + r_{,i}r_{,j}\right]$$





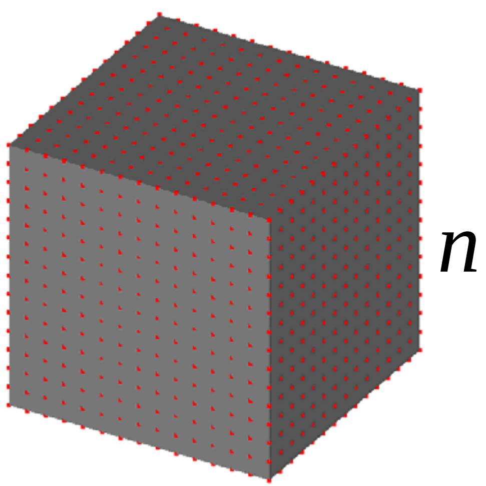
$$F(r) = -\frac{A}{r^m} + \frac{B}{r^n}$$

- Fast Multipole Method

  Memory complexity: $O(N)$; CPU complexity: $O(N)$

- Hierarchical Matrix and Adaptive Cross Approximation (ACA)

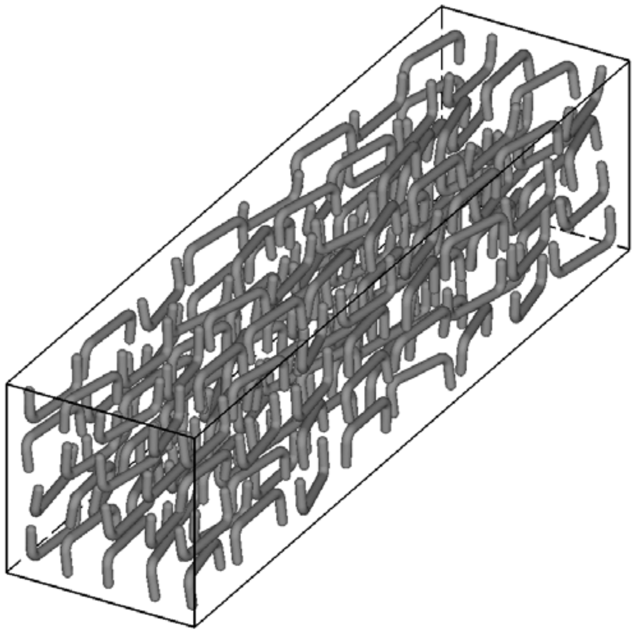
  Memory complexity: $O(N\log N)$; CPU complexity: $O(N\log^2 N)$

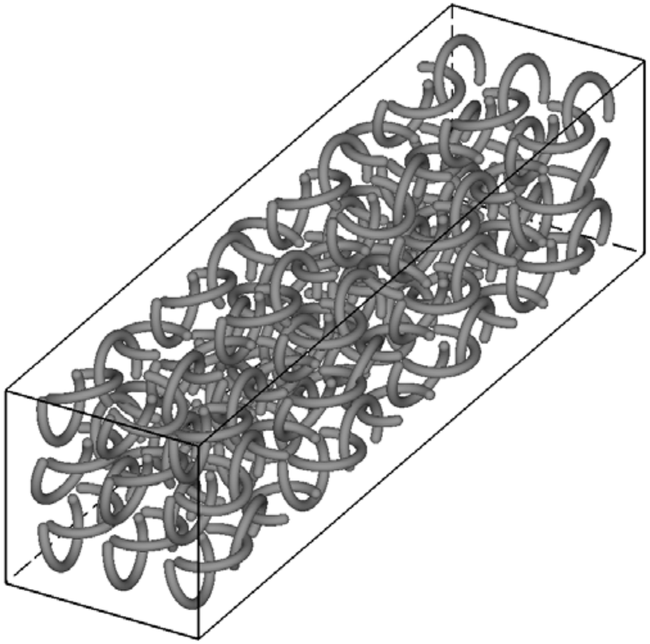


	Domain type methods (FEM, EFG, MLPG)	Boundary type methods (BEM, HdBNM)	Boundary type with linear complexity
Total degrees of freedom	$O(n^3)$	$O(n^2)$	$O(n^2)$
Memory requirement	$O(n^3)$	$O(n^4)$	$O(n^2)$
Time complexity	$O(n^3)$	$O(n^4)$	$O(n^2)$

Breakthrough in N-Body Problem

- CNT composite simulation

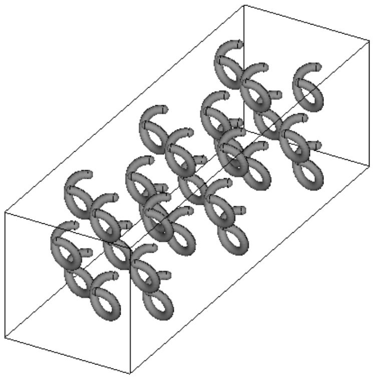




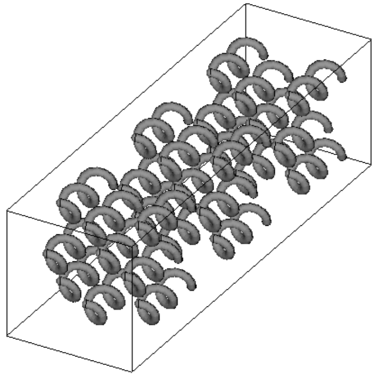
	κ	Nodes	Time (s)
HdBNM-FMM	1.337	165153	9776
BFM-ACA	1.353	165153	11945

	κ	Nodes	Time (s)
HdBNM-FMM	0.919	109314	5396
BFM-ACA	0.954	109314	6127

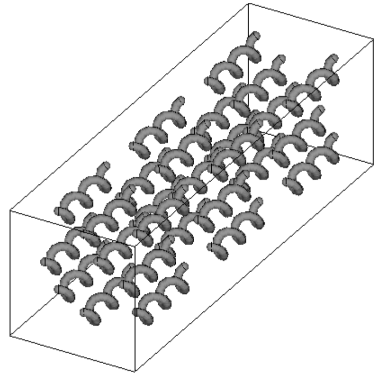
Breakthrough in N-Body Problem



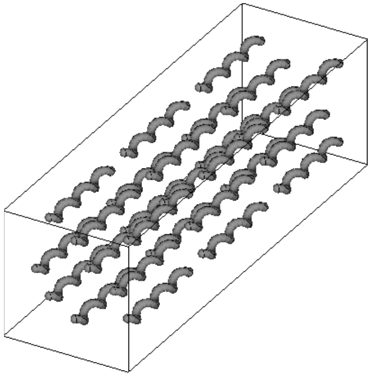
$\kappa = 0.4362$



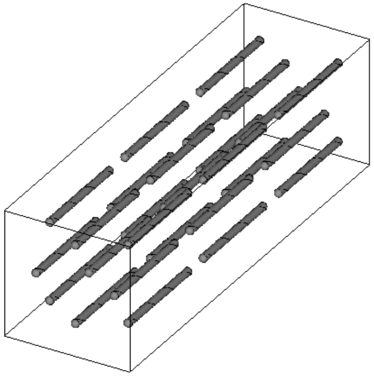
$\kappa = 0.6975$



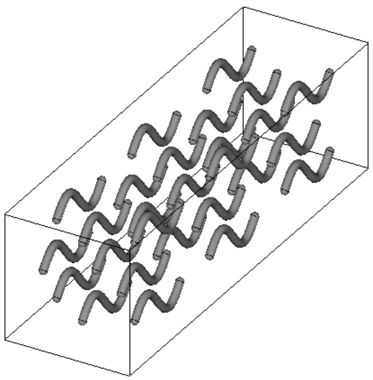
$\kappa = 0.8703$



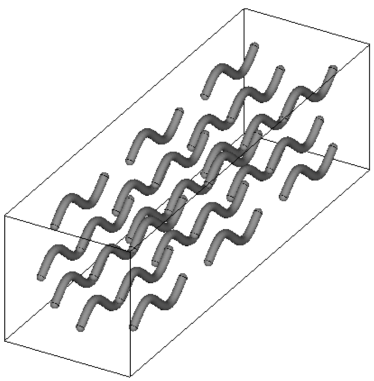
$\kappa = 0.9445$



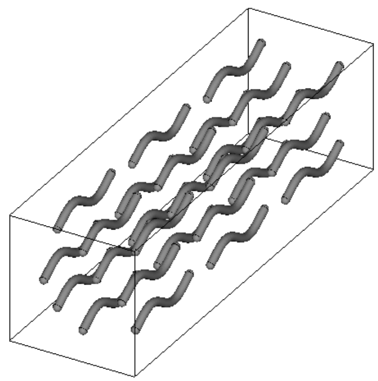
$\kappa = 0.9482$



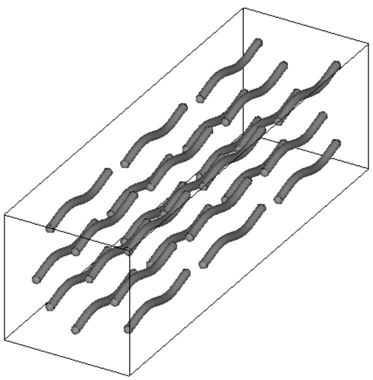
$\kappa = 0.6679$



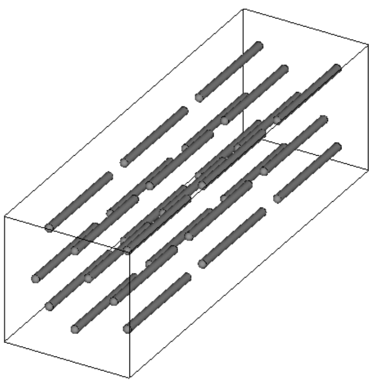
$\kappa = 0.7431$



$\kappa = 0.8257$



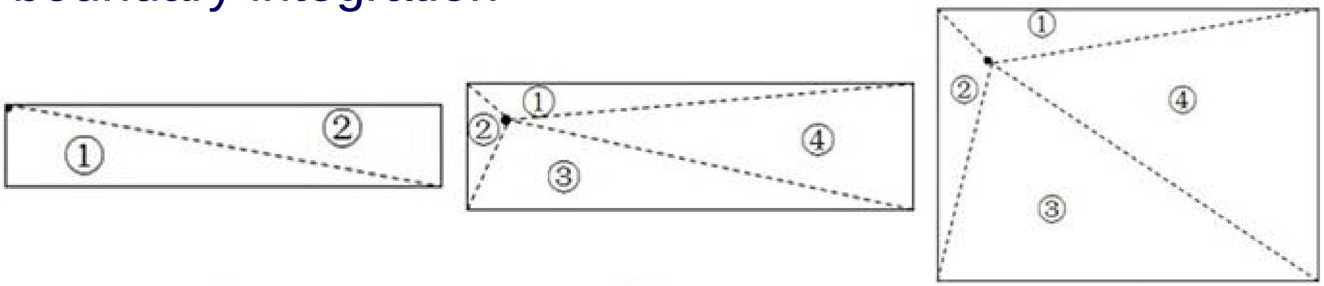
$\kappa = 0.9381$



$\kappa = 0.9482$

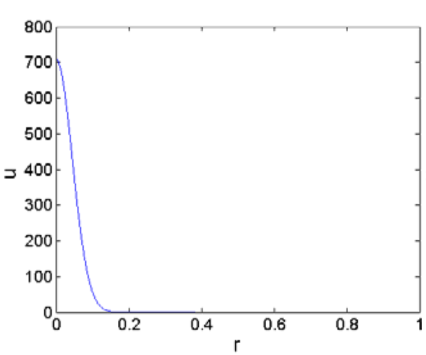
Spherical element subdivision method

- Singular boundary integration

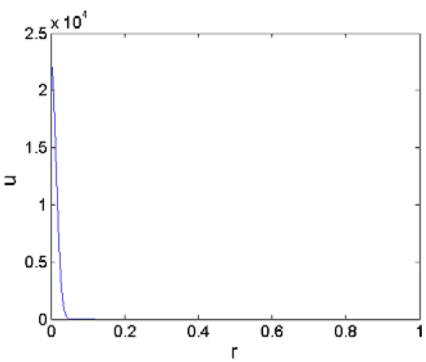


- Regular Volume integration

$$u^{s}=\frac{1}{(4\pi k\tau)^{1.5}}\exp\left(-\frac{r^{2}}{4k\tau}\right)$$

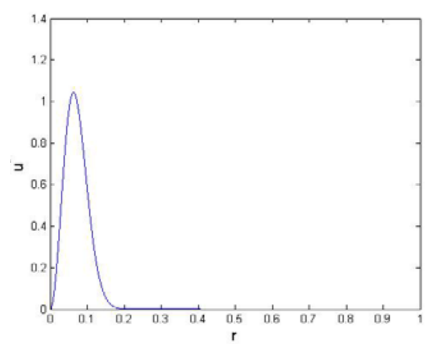


$\tau=0.001$

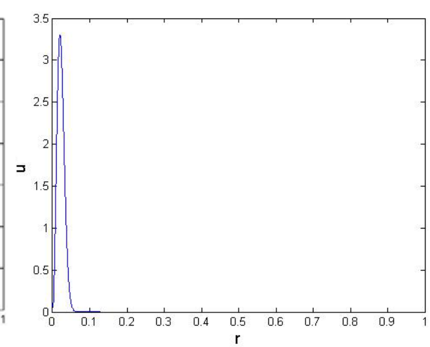


$\tau=0.0001$

u^s in Cartesian coordinate system



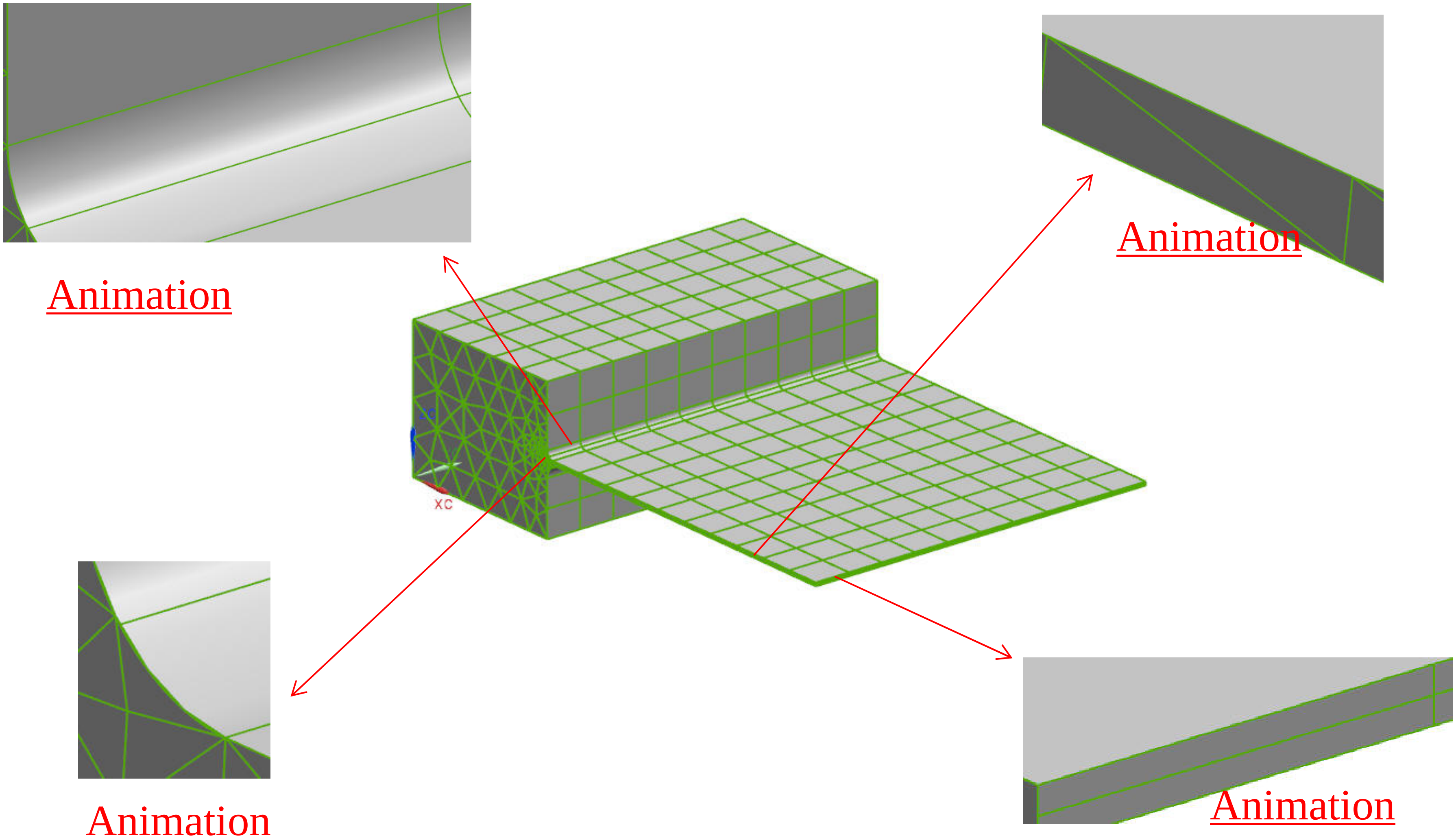
$\tau=0.001$



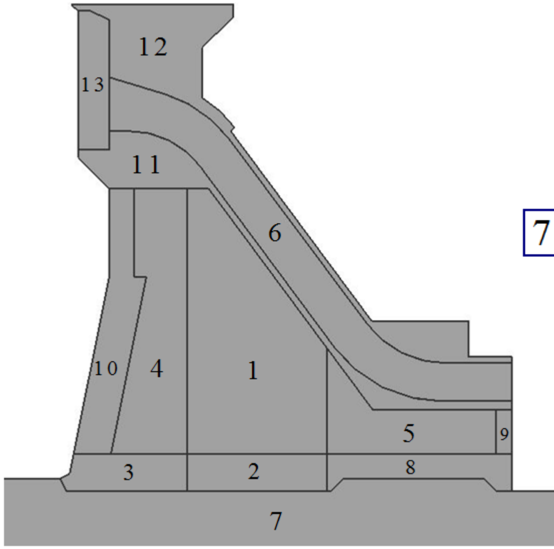
$\tau=0.0001$

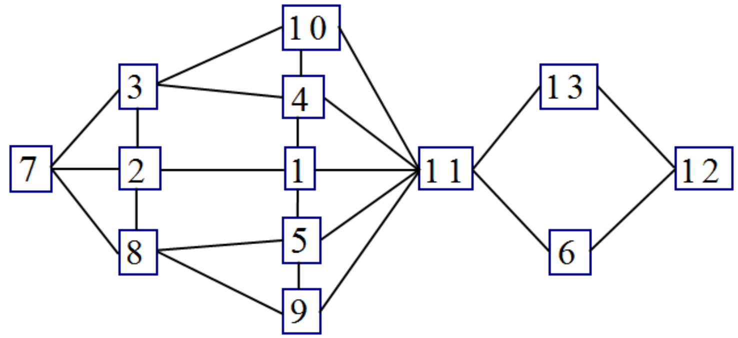
$\rho\, u^s$ in polar coordinate system

Spherical element subdivision method



Domain sequence optimization





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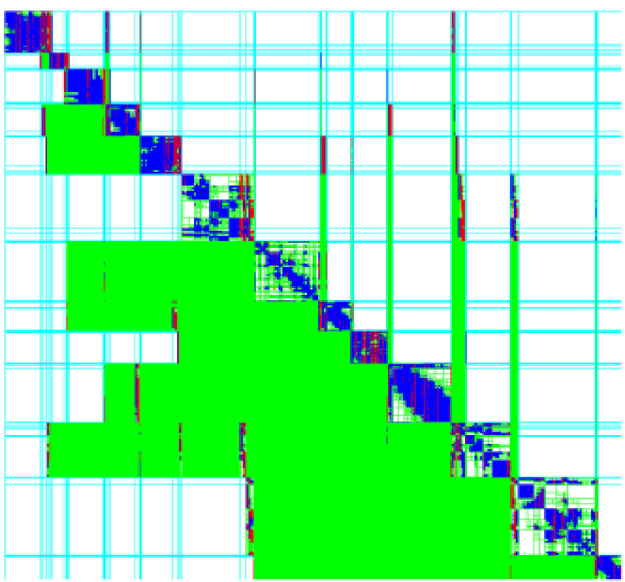
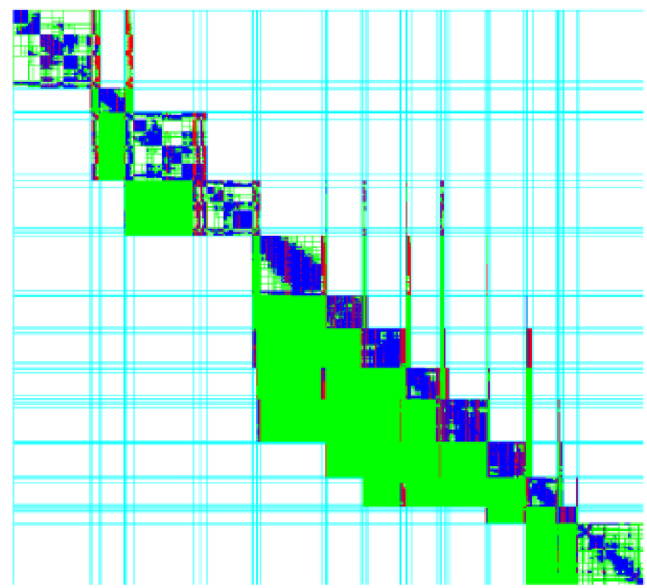
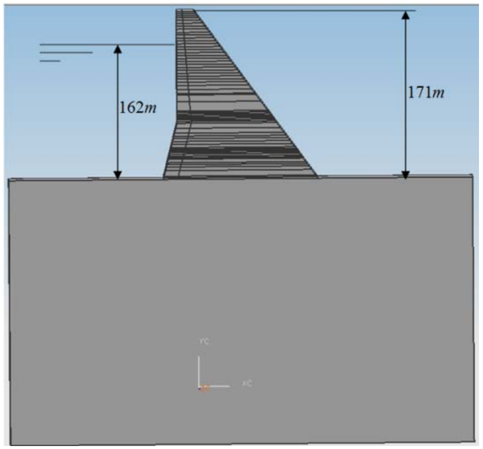




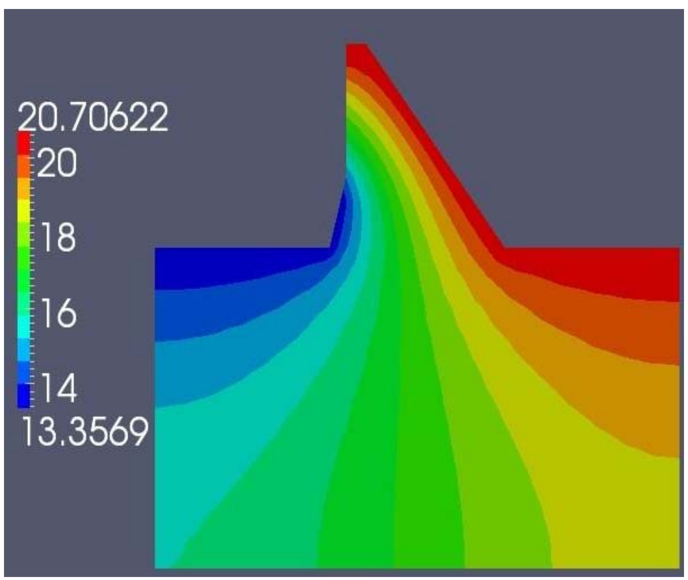
Numerical examples

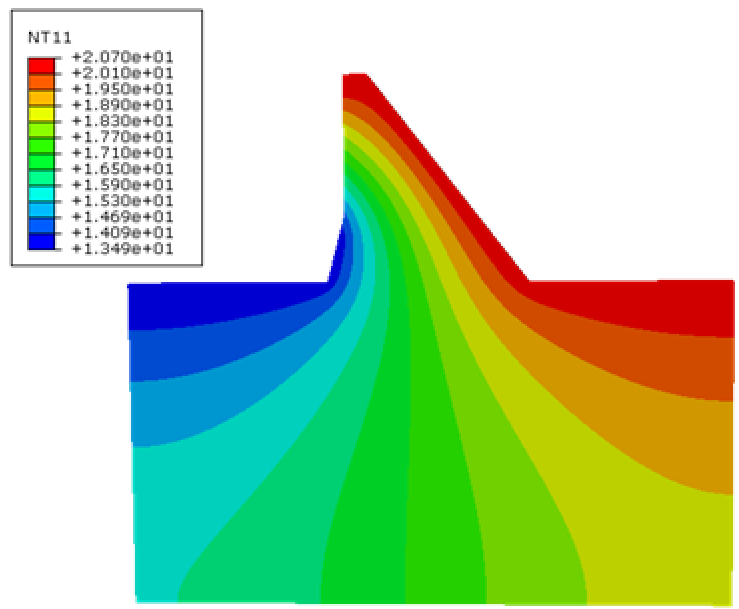
Steady state heat conduction



	c	ρ	κ
	$kJ/(kg\cdot{}^{0}C)$	kg/m^3	$kJ/(m\cdot h\,{}^{0}C)$
Dam	2400	0.9627	9.27
Base	2450	0.9627	8.776

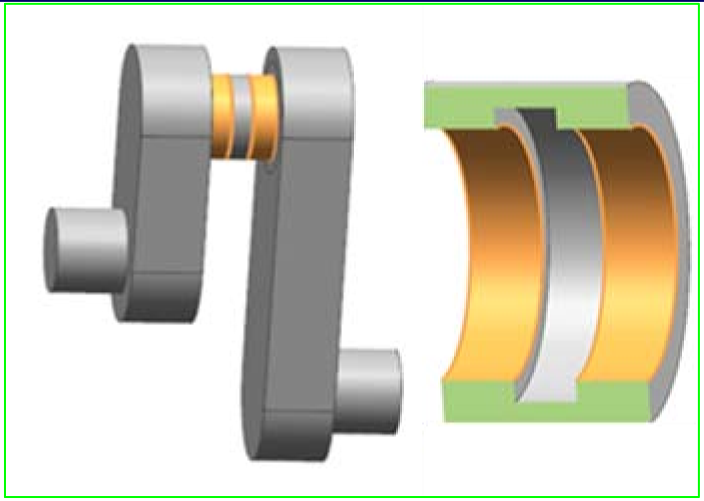
$$T_w = 20.7 - 0.0591572 \times h$$

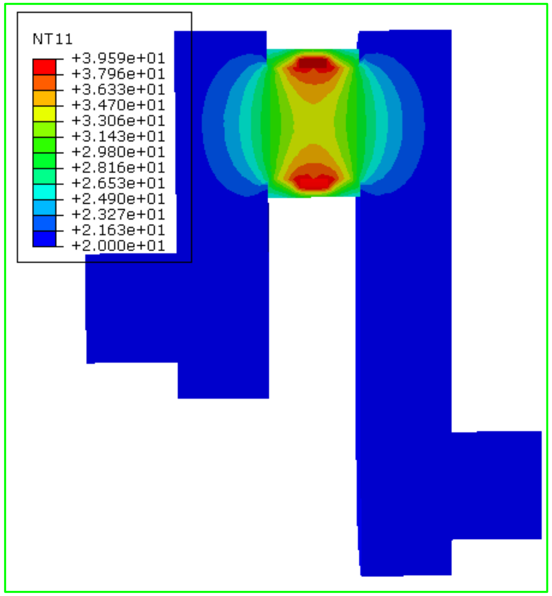


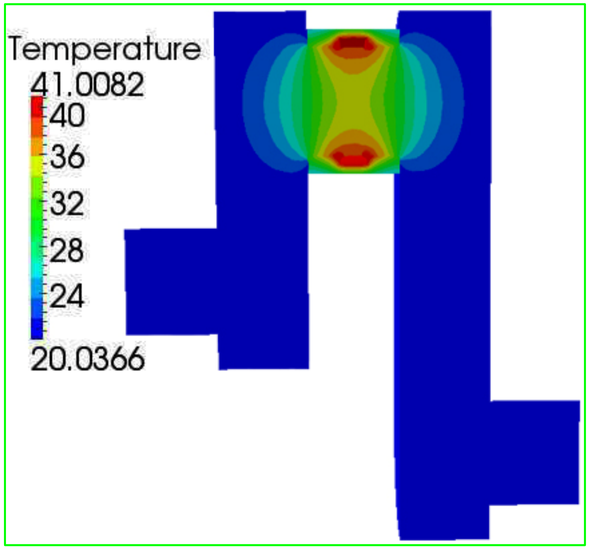


Steady state heat conduction

Crankshaft of an engine

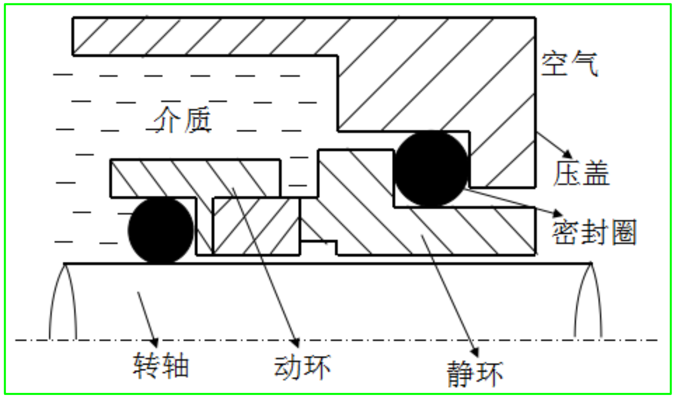


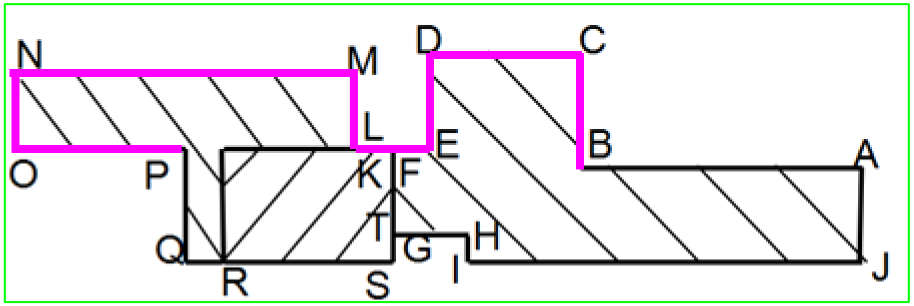


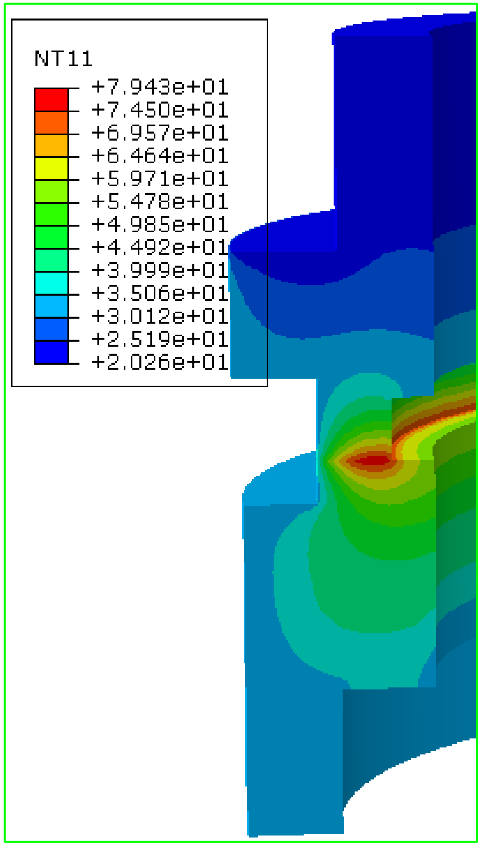


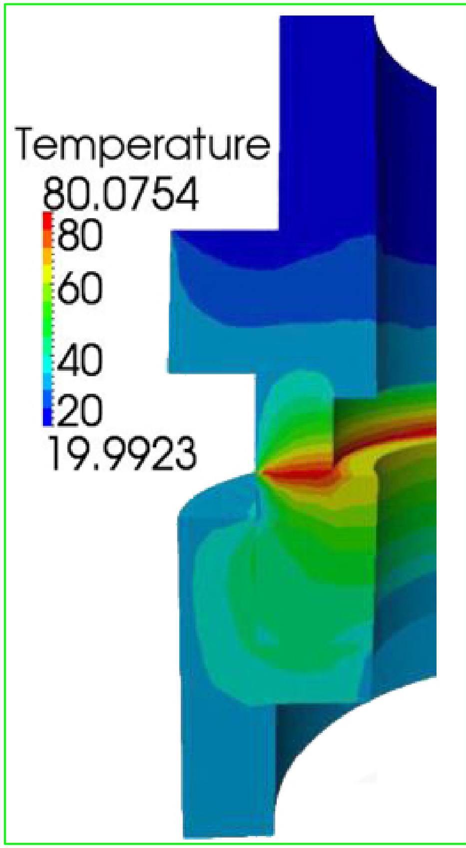
Steady state heat conduction

Sealing ring

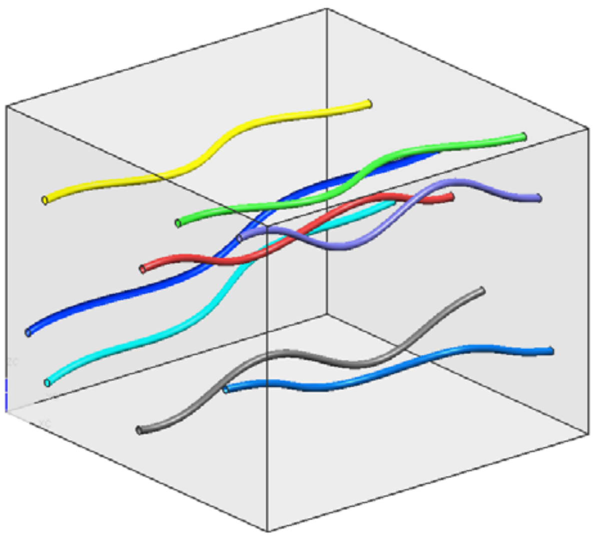


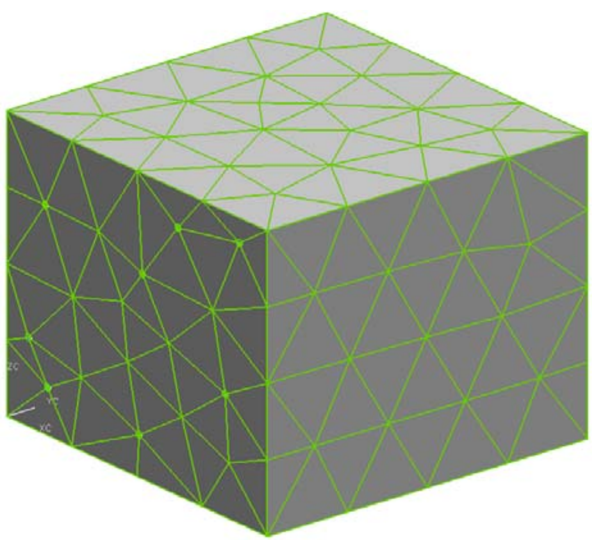


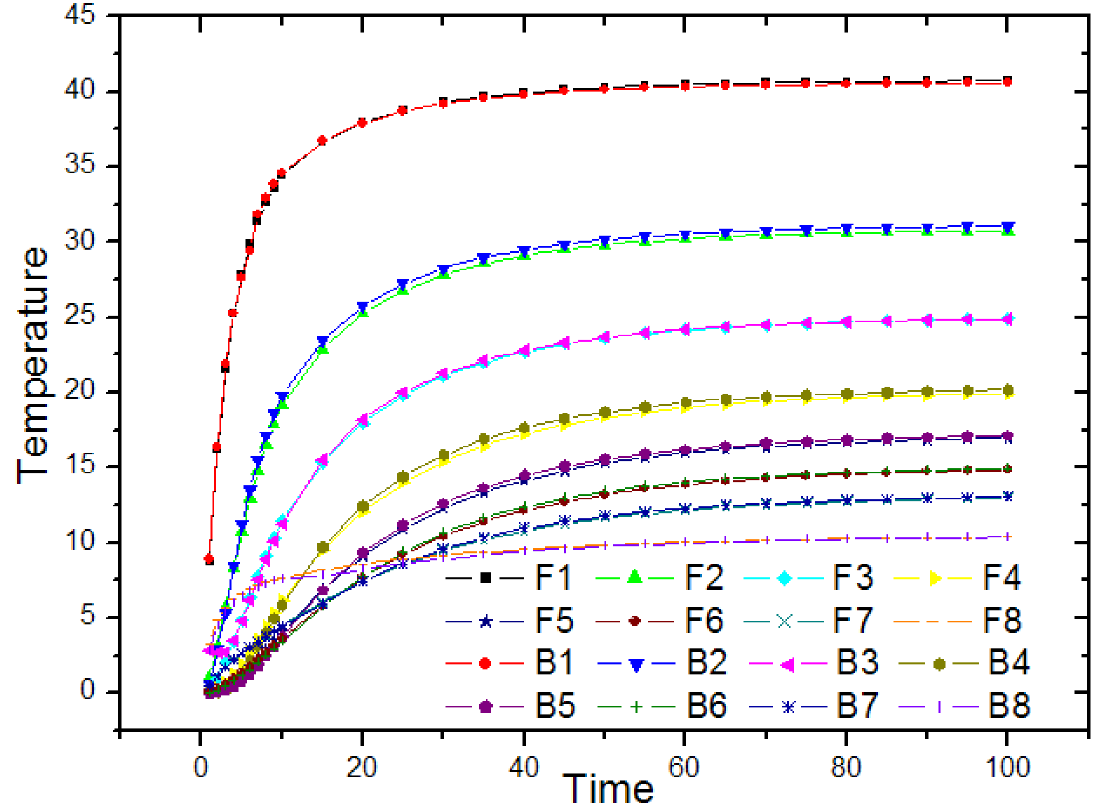




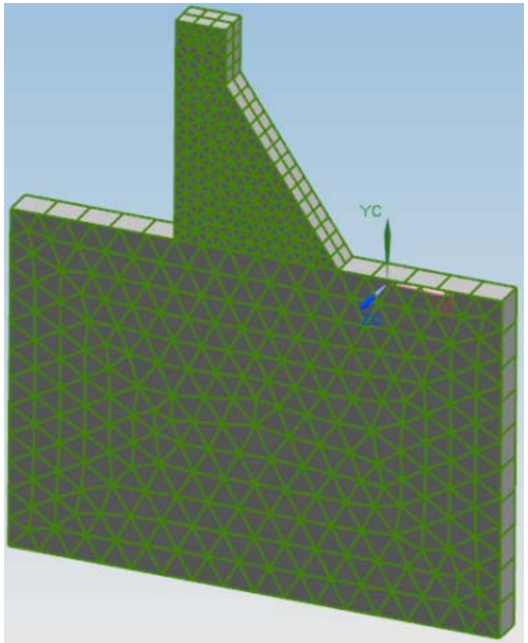
Transient heat conduction

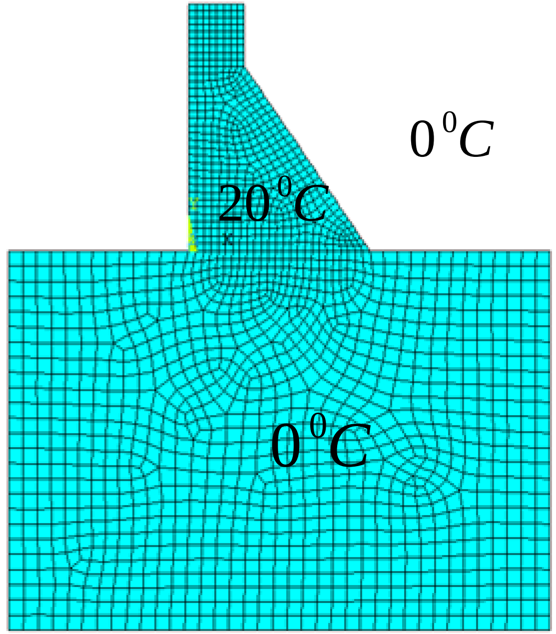


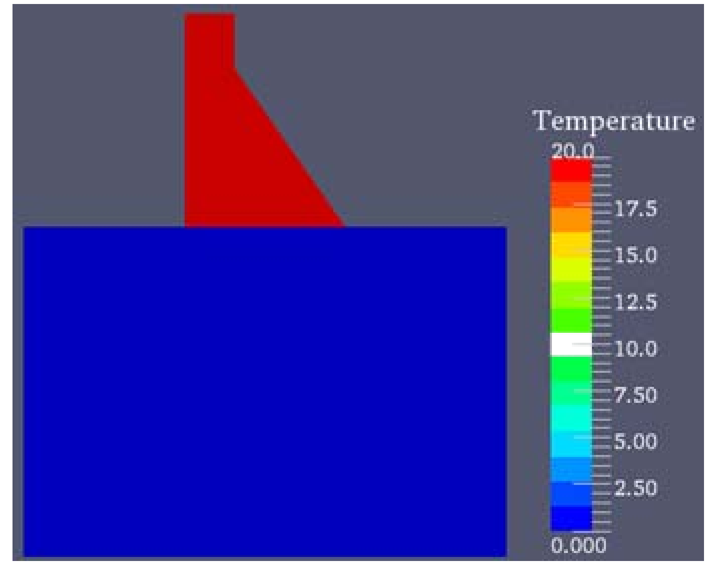


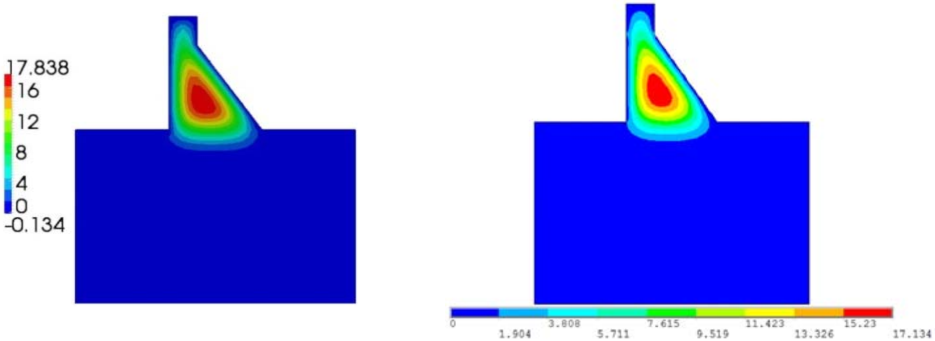


Natural cooling process

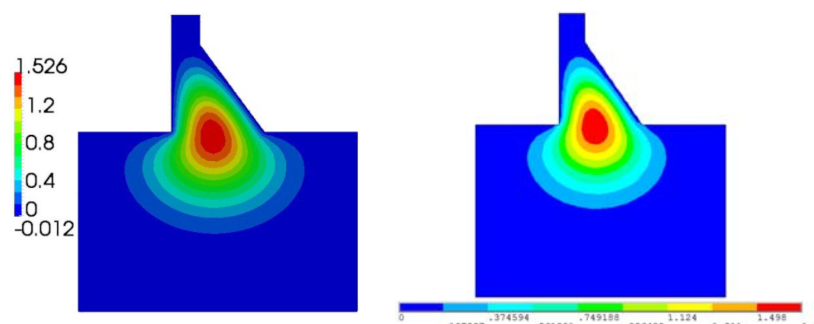








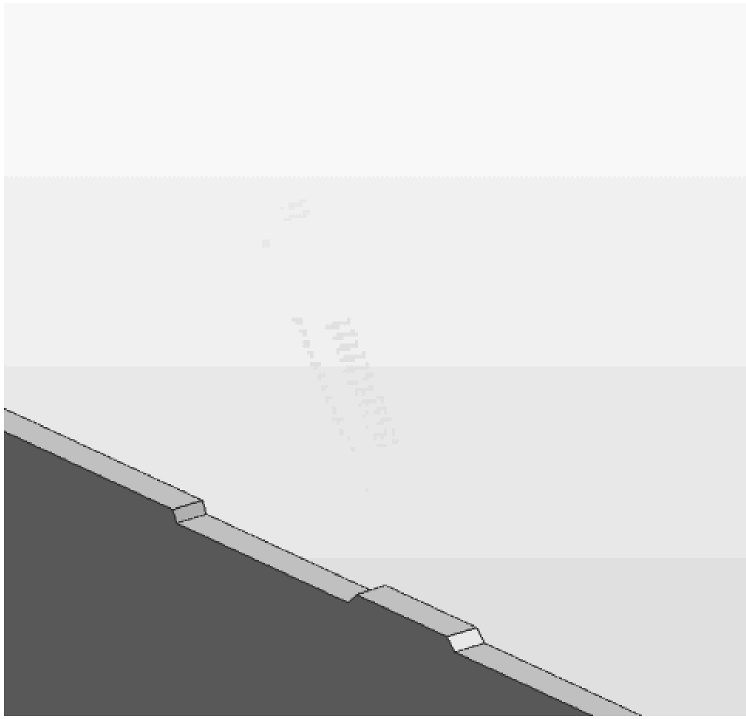
1 year



8 year

The Problem (RCC Dam)

- Construction of roller compacted concrete(RCC) dams



Hydration of Cement → Heat

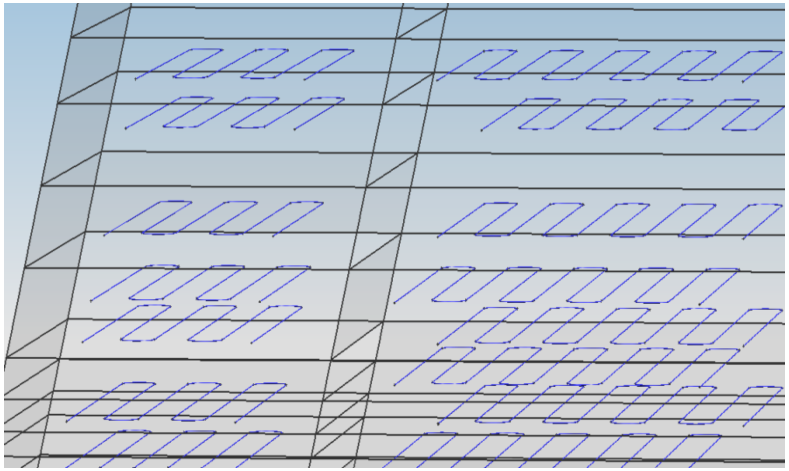
Low conductivity of Concrete

→ Thermal Stress (Visco-elastic) → Cracks

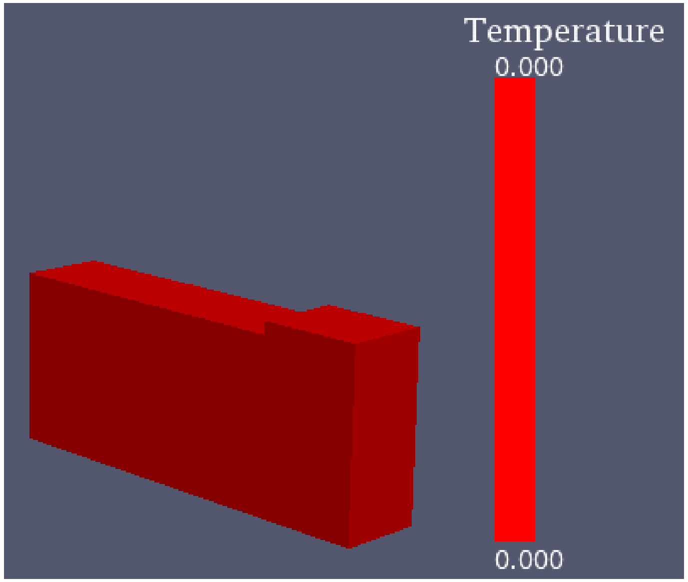
The Problem (RCC Dam)

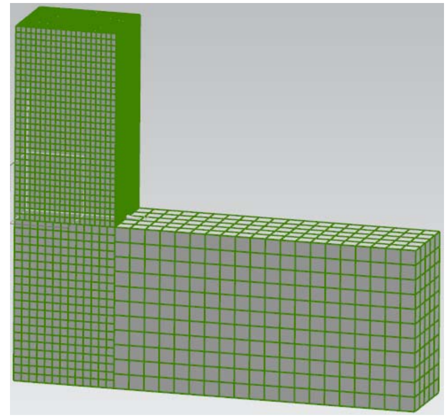
- Factors that affect temperature distribution, evolution and the induced thermal stresses
  - Concrete properties (heat of hydration, conductivity, creep and shrinkage, etc.)
  - Ambient conditions (foundation temperature, solar radiation and wind speed, etc.)
  - RCC placing temperature, the thickness of lift and the casting schedule of the concrete

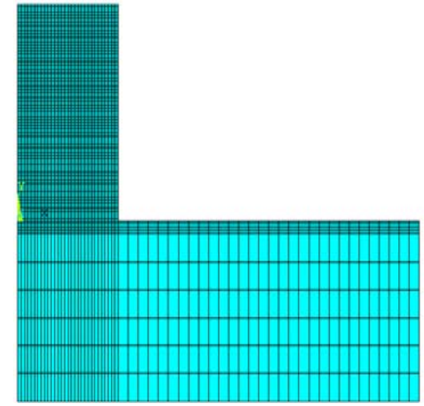
- Measures against cracks
  - Prolong Placement period
  - Embedding cooling water pipes



Lifting Layers

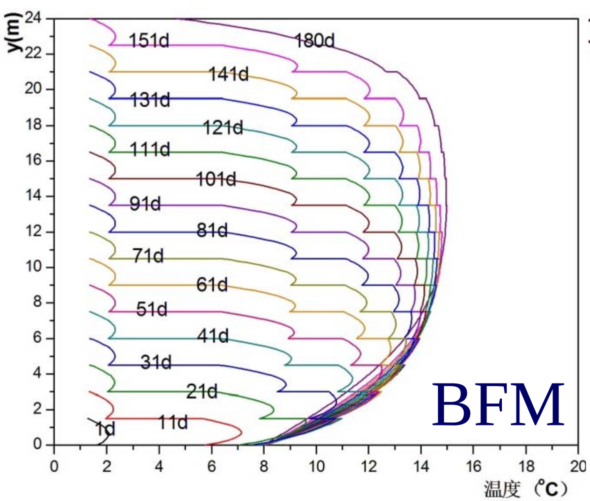


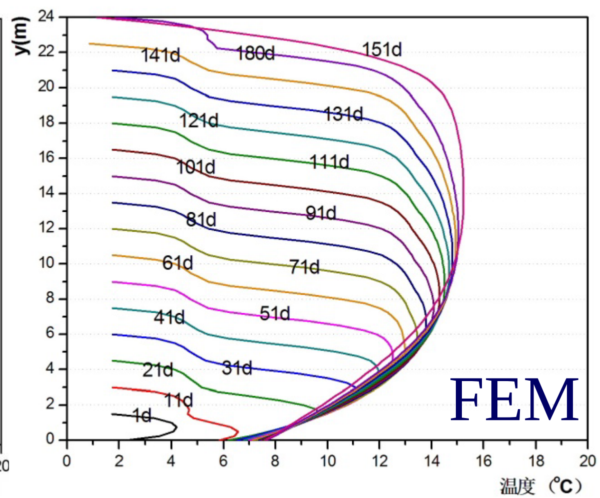




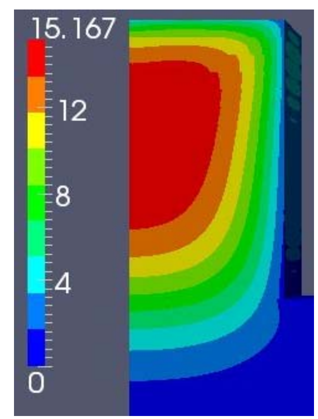
$$\theta(\tau)=\frac{25.0\tau}{4.5+\tau}(℃)$$

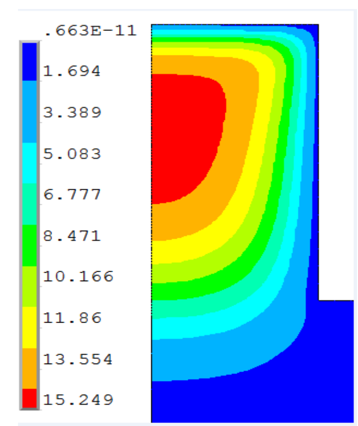
$$\lambda\frac{\partial T}{\partial n}=-\beta(T-T_a)$$





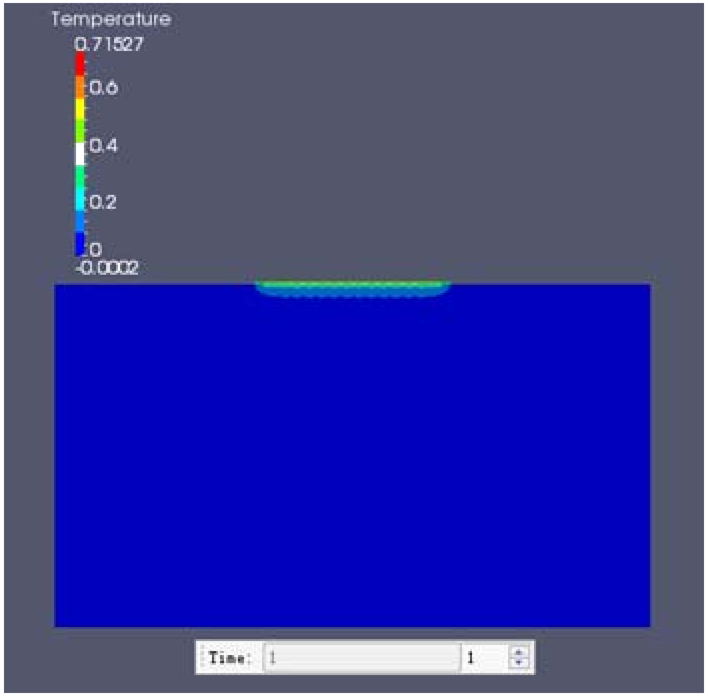
Temperature evolution

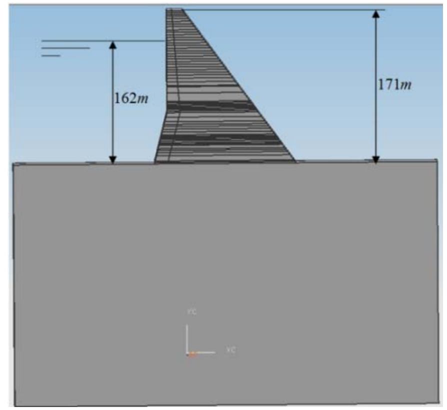




Thermal state at day 180

Real dam simulation





Time span 9 years

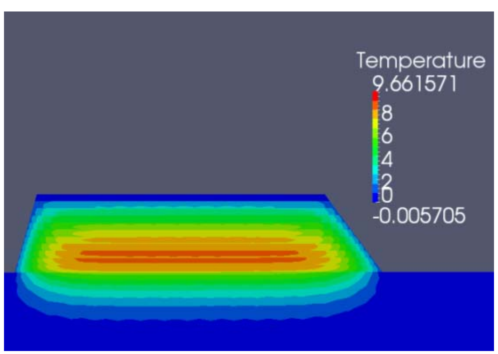
1260 time steps

Heat conductivity: $10\,kJ/(m\cdot h^{0}C)$

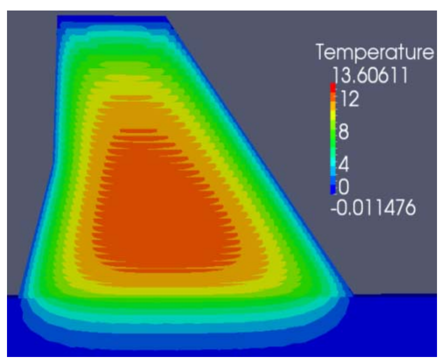
Heat diffusivity: $0.004\ m^2/h$

Ambient temperature: $T_a = 0(℃)$

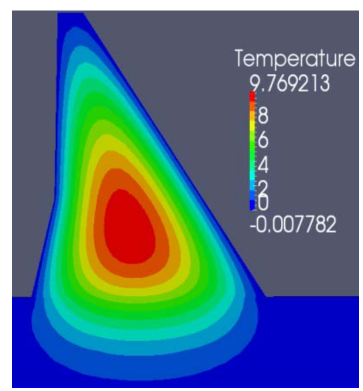
Adiabatic temperature rise: $\theta(\tau) = \dfrac{25.0\tau}{4.5+\tau}(℃)$



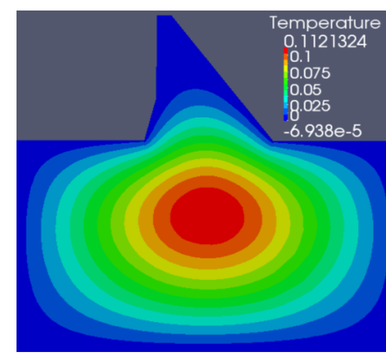
At hour 100



At hour 400



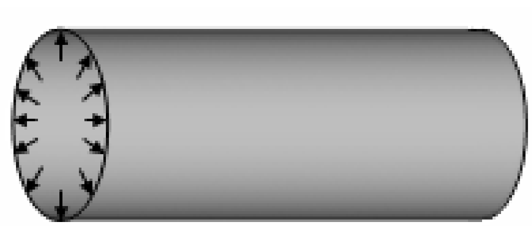
At day 38

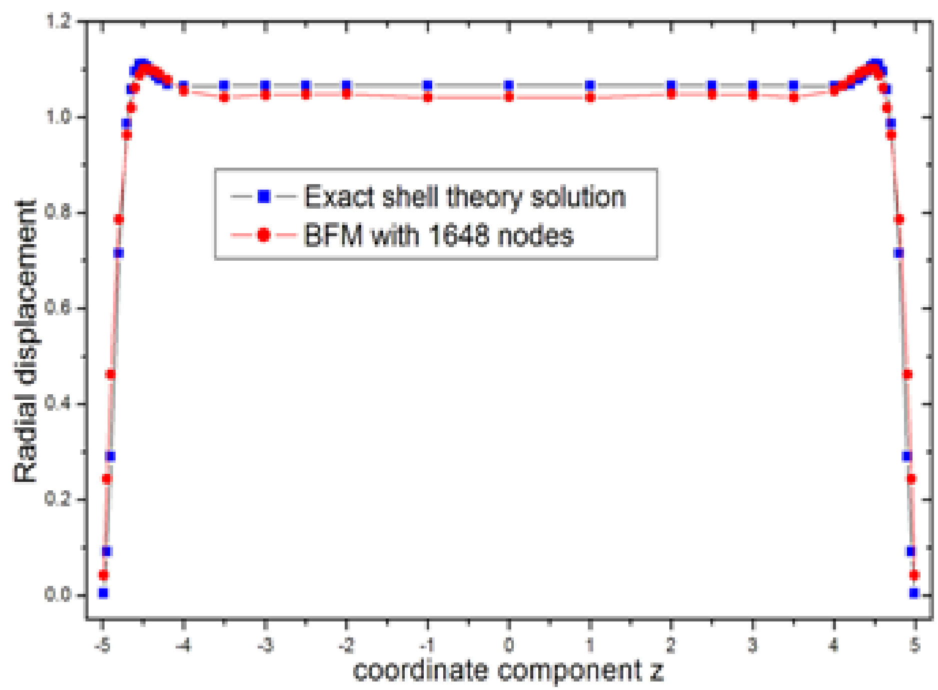


At year 9

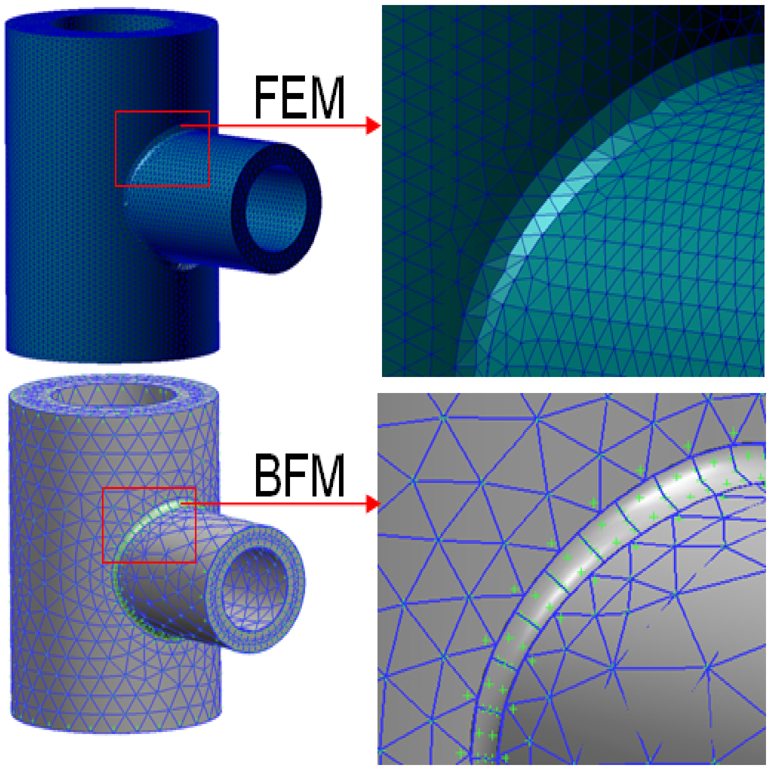
Stress concentration

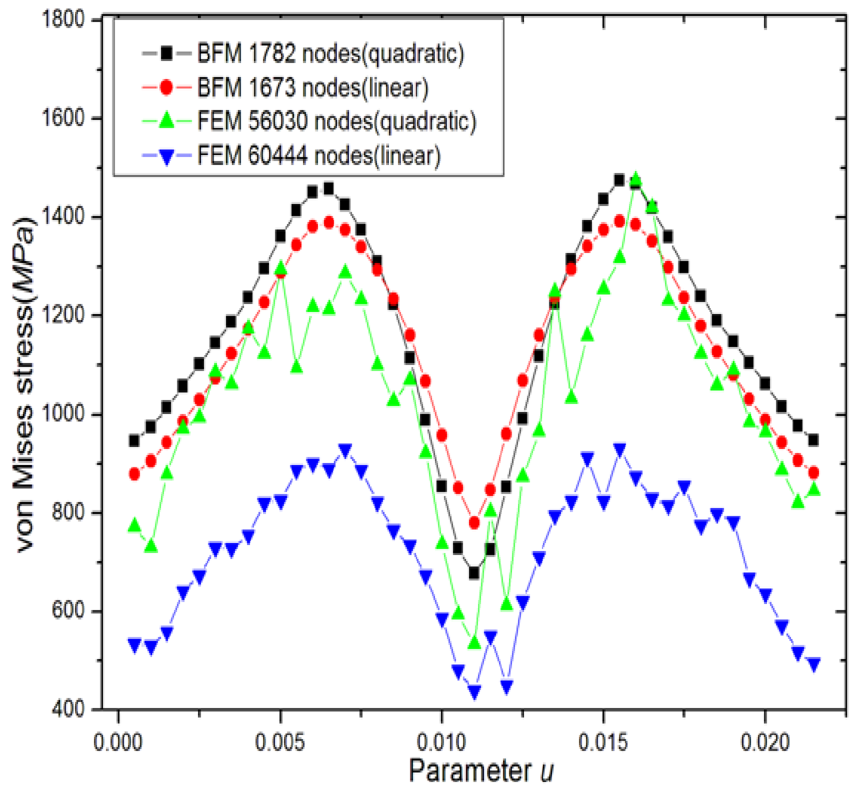
- Very thin shell under pressure





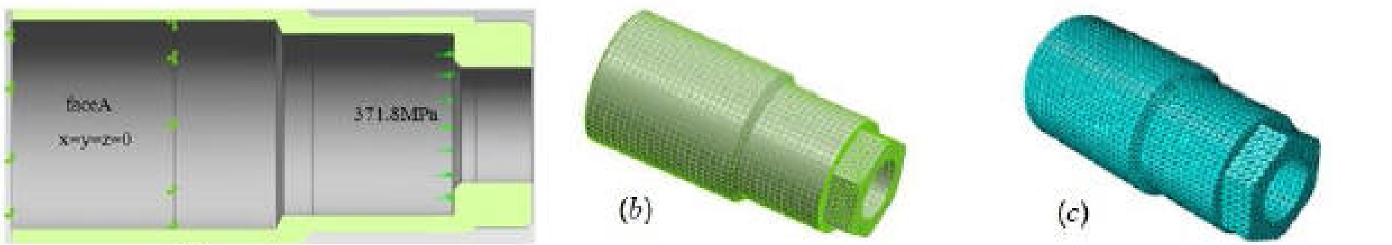
■ Manifold with fillet





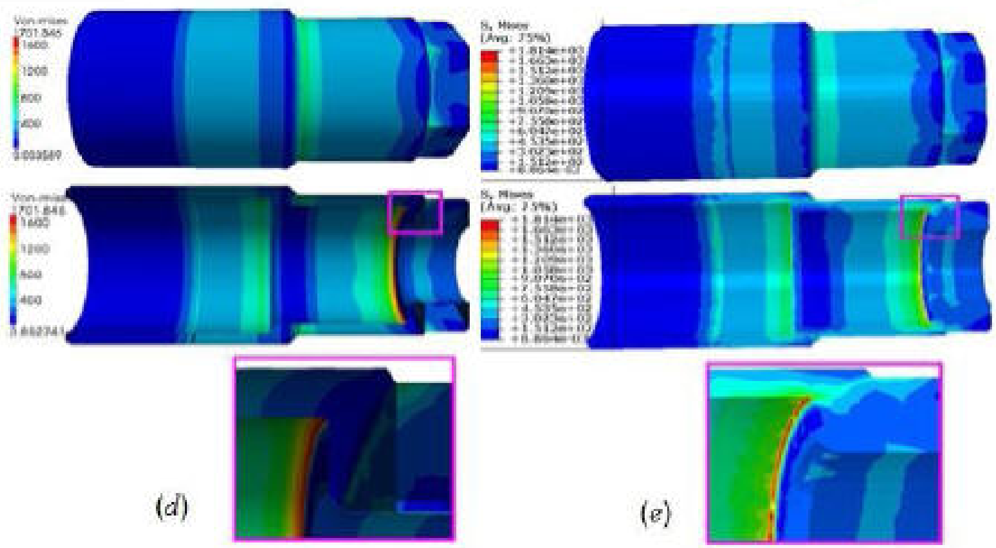
Stress concentration

- Nozzle cap nut



(a) (b) (c)

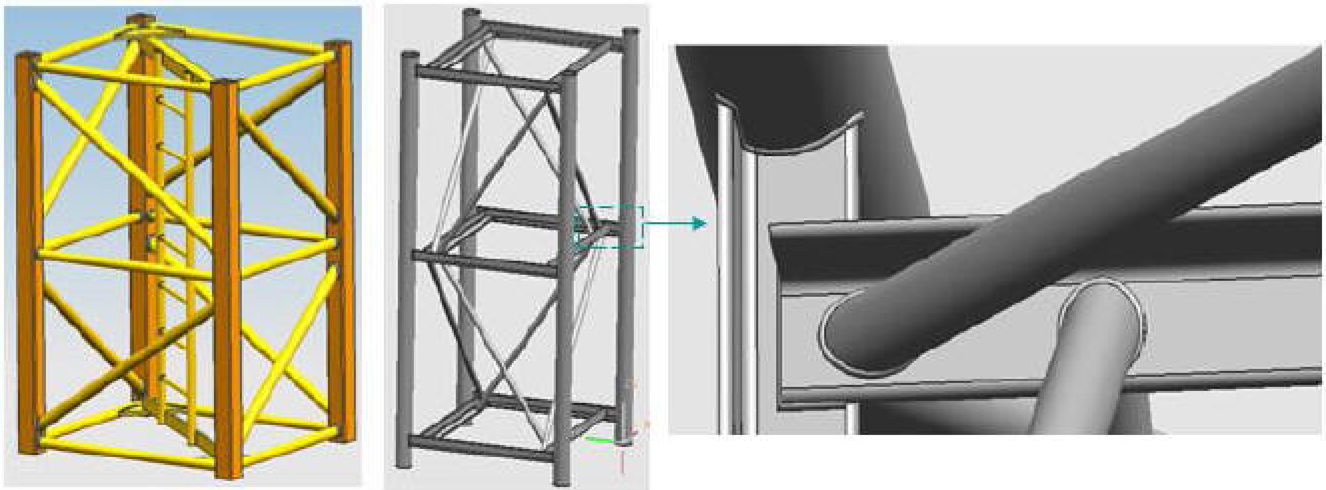
算法	节点数目	最大应力(MPa)
BFM	5422	1710.15
	8014	1701.85
	14455	1701.85
FEM	35098	1698
	89548	1762
	136733	1814

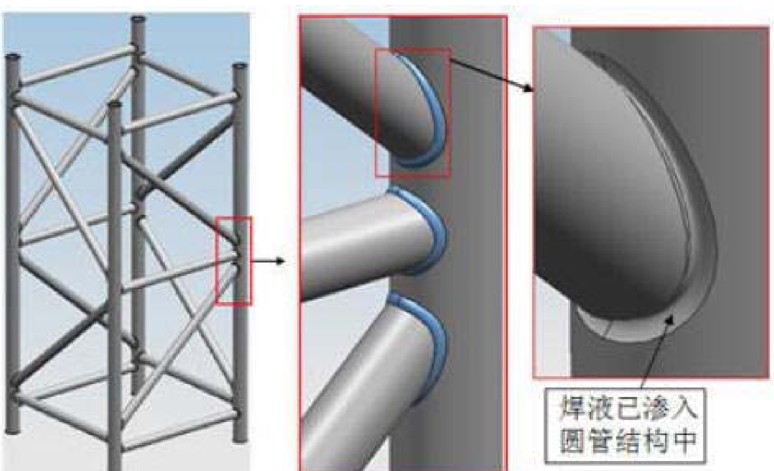


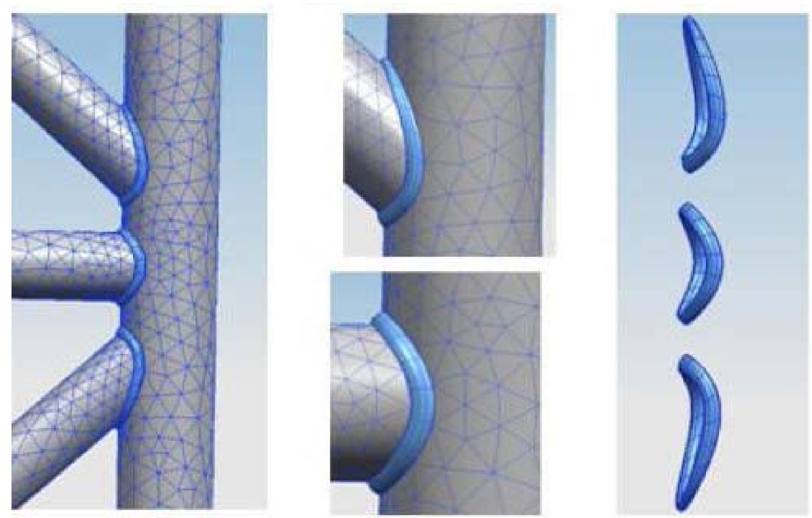
(d) (e)

Stress concentration

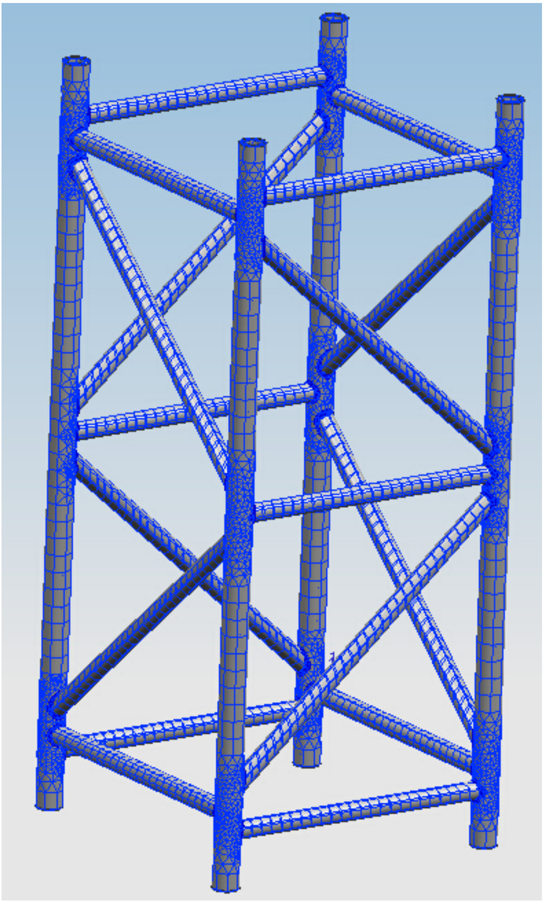
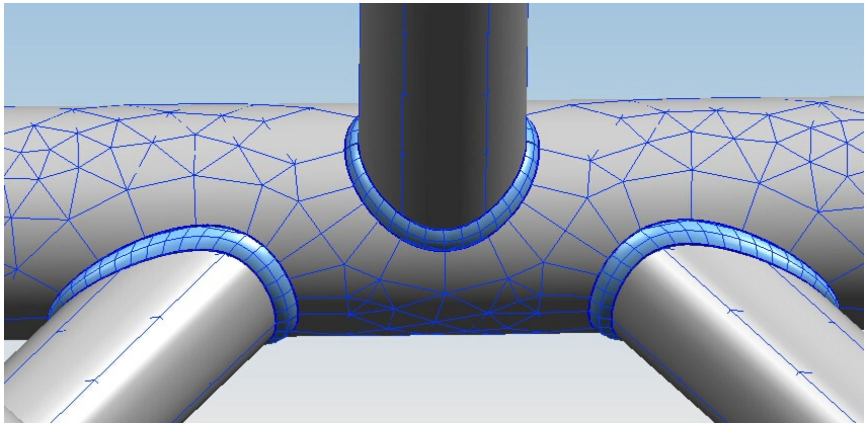
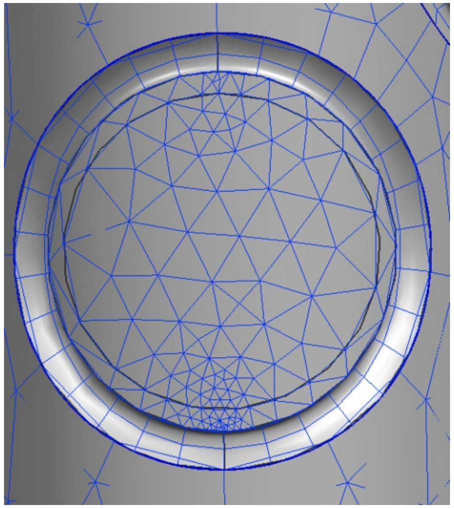
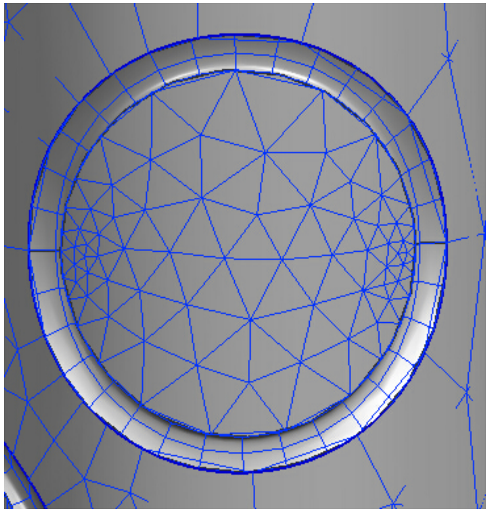
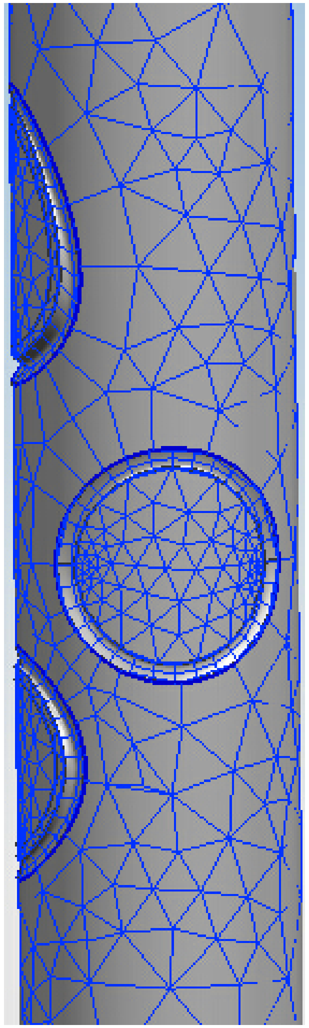
- Welded Steel Frame



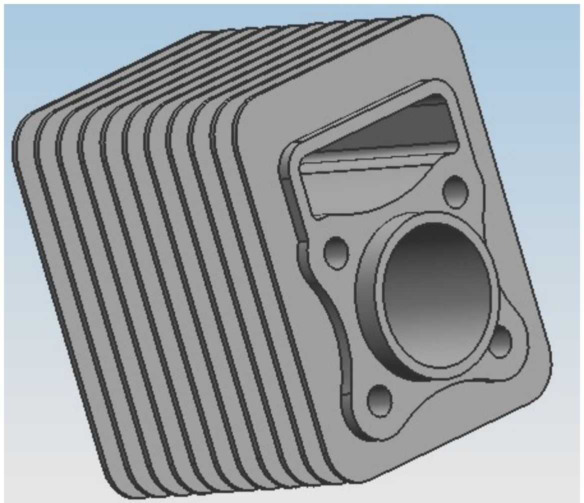


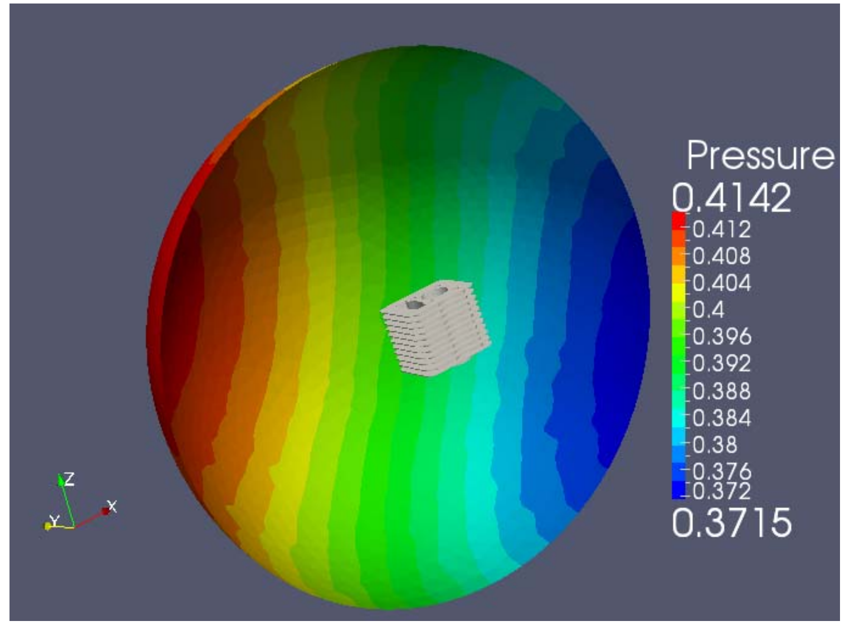


Stress concentration

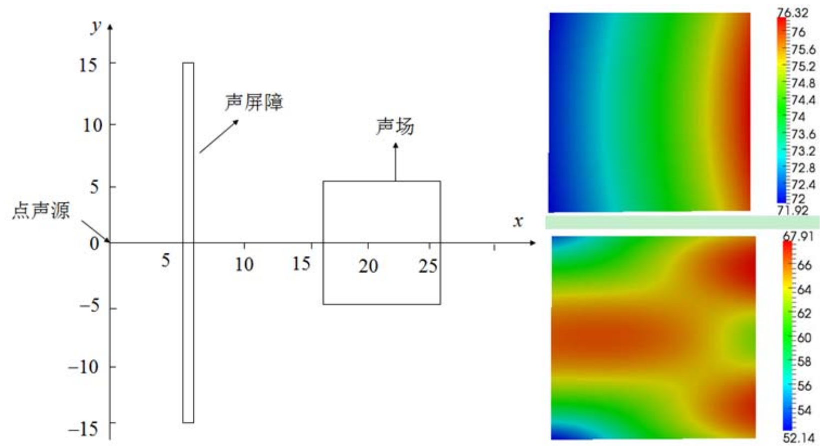
Acoustic problem



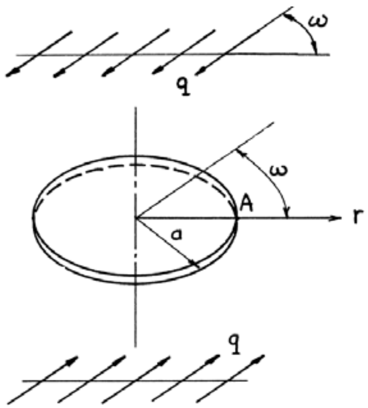


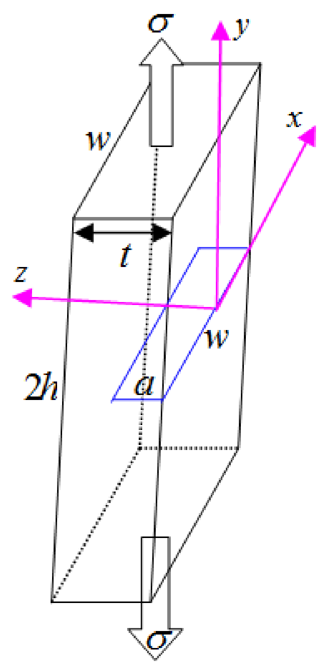
Radius=5.0
Wave number=1

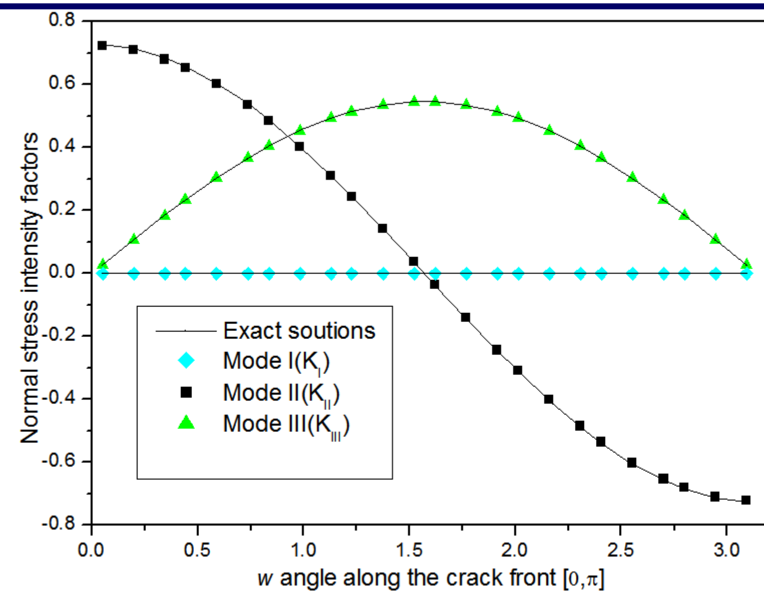
20760 triangular elements

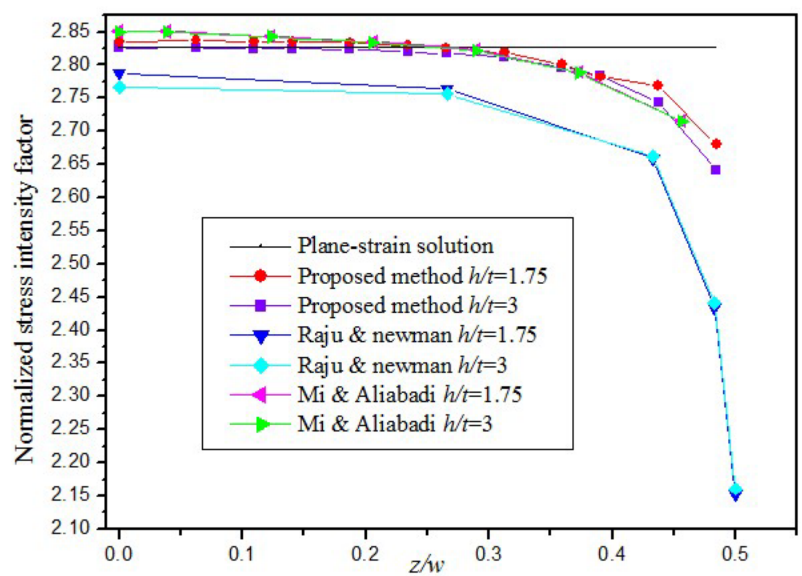


Fracture problem









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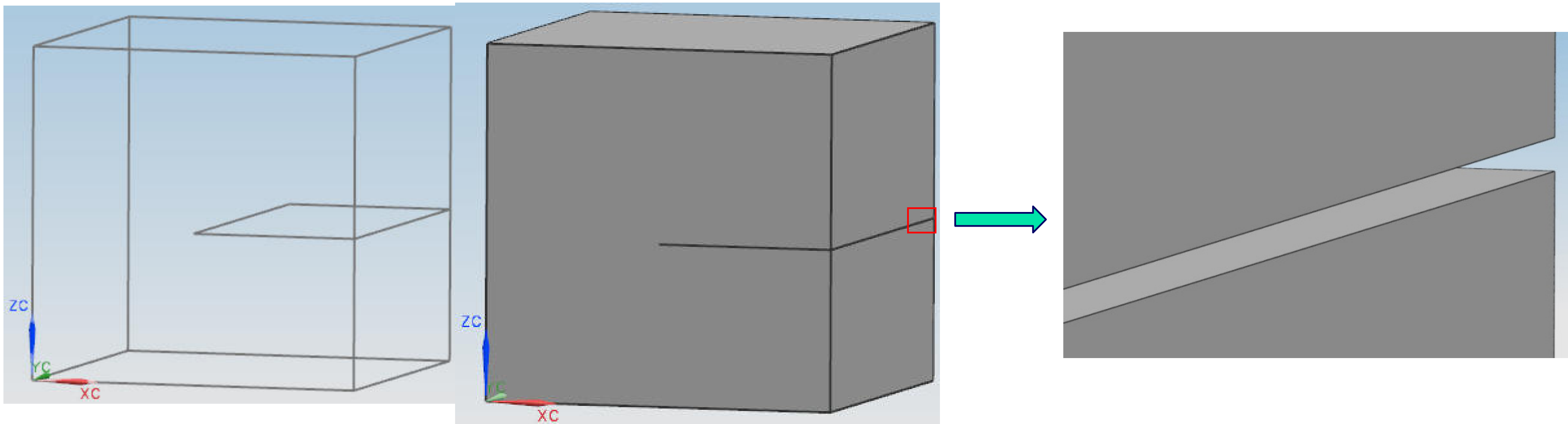
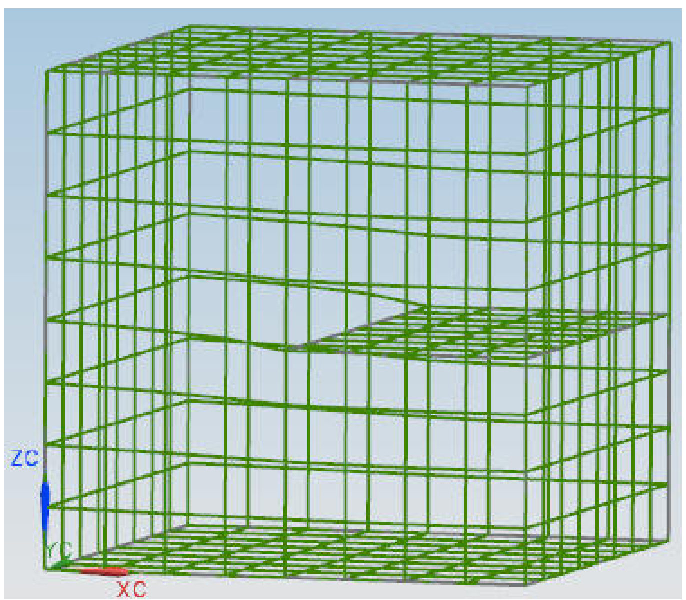
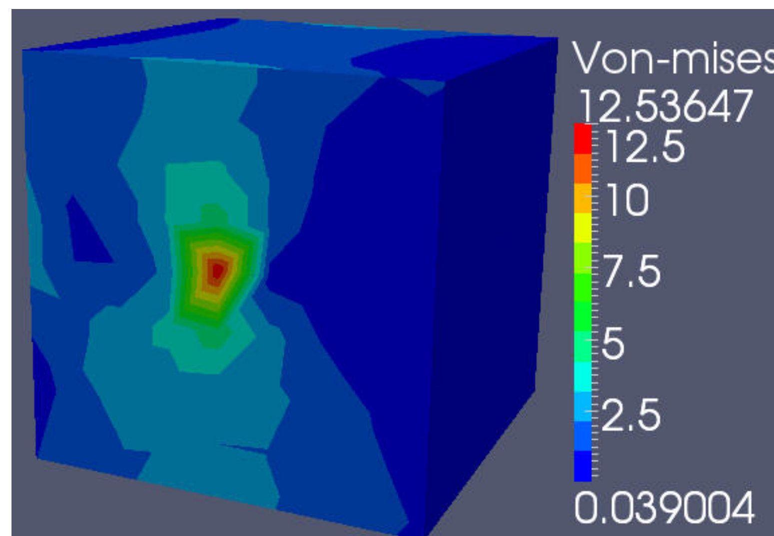
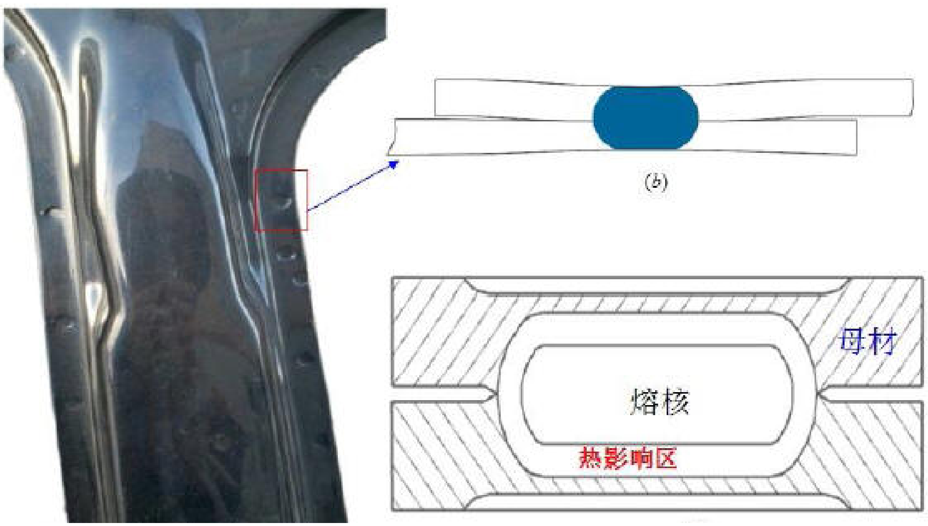


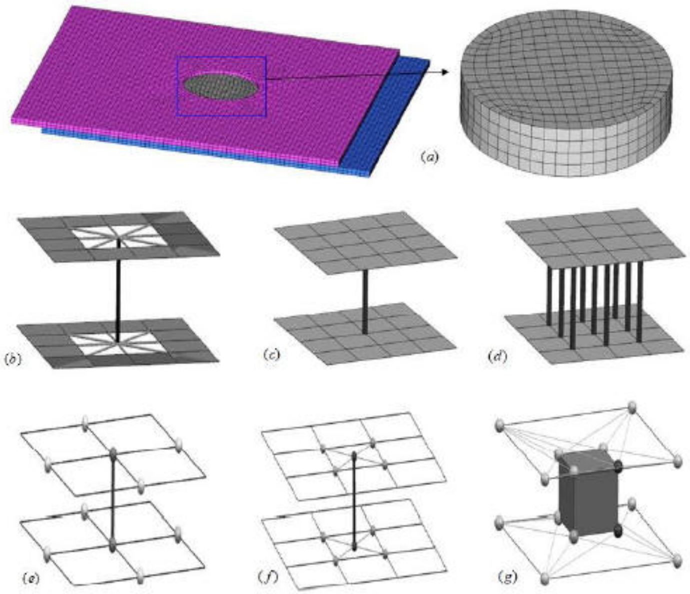




Fracture problem

- Resistance Spot Welding





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Pronunciation:

“5aCAE” ==“吾爱CAE”

Meaning:

“吾爱CAE” == “I love CAE !!!”